



**School of Electronic
and Communications
Engineering**

Analog Multipliers

Recommended Text: Gray, P.R. & Meyer. R.G.,
Analysis and Design of Analog Integrated Circuits (3rd
Edition), Wiley (1992) pp. 667-681

Introduction

- ❖ Nonlinear operations on continuous-valued analog signals are often required in instrumentation, communication, and control-system design.
- ❖ These operations include
 - rectification,
 - modulation,
 - demodulation,
 - frequency translation,
 - multiplication, and
 - division.
- ❖ In this chapter we analyze the most commonly used techniques for performing **multiplication and division** within a monolithic integrated circuit

- ❖ In analog-signal processing the need often arises for a circuit that takes two analog inputs and produces an output proportional to their product.
- ❖ Such circuits are termed *analog multipliers*.
- ❖ In the following sections we examine several analog multipliers that depend on the **exponential** transfer function of bipolar transistors .

The Emitter-Coupled Pair as a Simple Multiplier

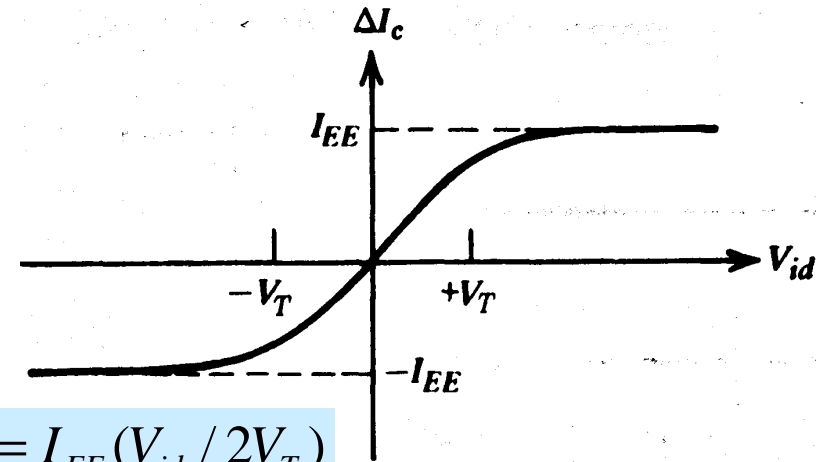
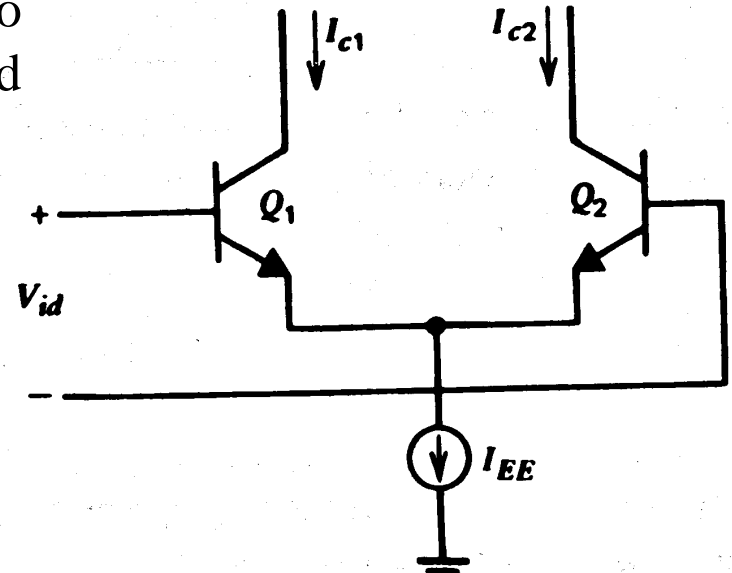
- ❖ The emitter-coupled pair, was shown in to produce output currents that were related to the differential input voltage by :

$$I_{c1} = \frac{I_{EE}}{1 + \exp(-V_{id} / V_T)} \quad I_{c2} = \frac{I_{EE}}{1 + \exp(V_{id} / V_T)}$$

$$\Delta I_c = I_{c1} - I_{c2} = I_{EE} \tanh(V_{id} / 2V_T)$$

- ❖ This relationship is plotted => and shows that the emitter-coupled pair by itself can be used as a primitive multiplier.

$$\text{or assuming } (V_{id} / 2V_T) \ll 1, \Rightarrow \Delta I_c = I_{EE} (V_{id} / 2V_T)$$

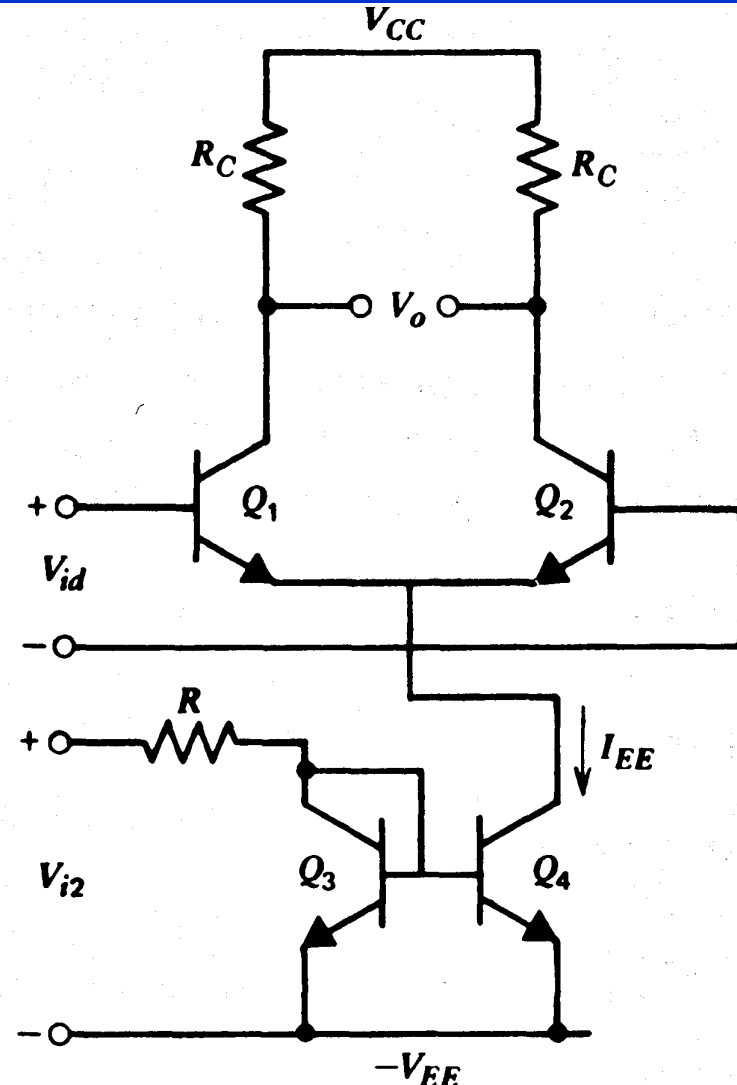


- ❖ The current I_{EE} is actually the bias current for the emitter-coupled pair.
- ❖ With the addition of more circuitry, we can make I_{EE} proportional to a second input signal.
- ❖ Thus we have

$$I_{EE} \cong K_o (V_{i2} - V_{BE(on)})$$

- ❖ The differential output current of the emitter-coupled pair can be calculated to give

$$\Delta I_c \cong \frac{K_o V_{id} (V_{i2} - V_{BE(on)})}{2V_T}$$



Two-Quadrant restriction

- ❖ Thus we have produced a circuit that functions as a multiplier under the assumption that V_{id} is small, and that V_{i2} is greater than $V_{BE(on)}$.
- ❖ The latter restriction means that the multiplier functions in only two quadrants of the $V_{id} - V_{i2}$ plane, and this type of circuit is termed a **two-quadrant** multiplier.
- ❖ The restriction to two quadrants of operation is a severe one for many communications applications, and most practical multipliers allow **four-quadrant** operation.
- ❖ The **Gilbert multiplier cell**, shown, is a modification of the emitter-coupled cell, which allows four-quadrant multiplication.

Gilbert multiplier cell

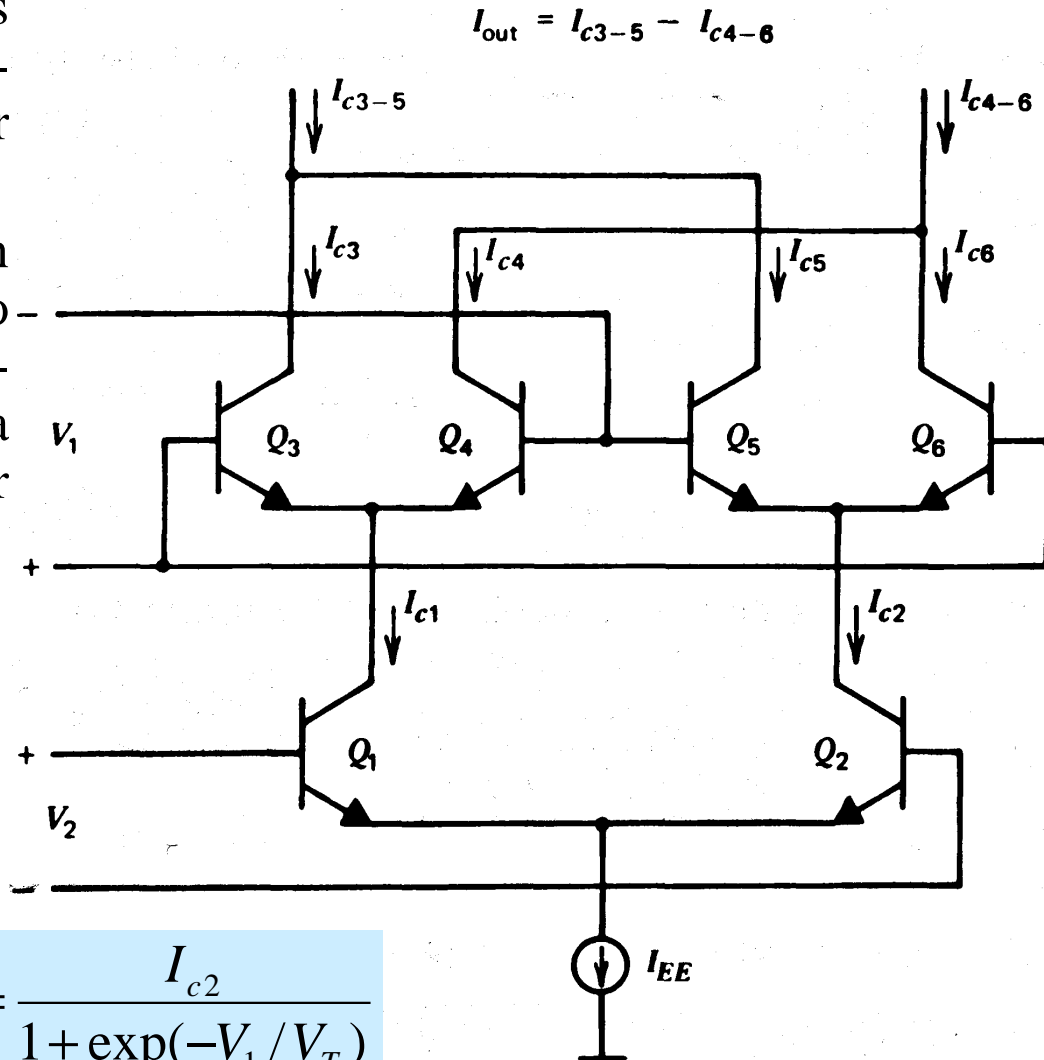
- ❖ The Gilbert multiplier cell is the basis for most integrated-circuit balanced multiplier systems.
- ❖ The series connection of an emitter-coupled pair with two cross-coupled, emitter-coupled pairs produces a particularly useful transfer characteristic,

$$I_{c3} = \frac{I_{c1}}{1 + \exp(-V_1 / V_T)}$$

$$I_{c4} = \frac{I_{c1}}{1 + \exp(V_1 / V_T)}$$

$$I_{c5} = \frac{I_{c2}}{1 + \exp(V_1 / V_T)}$$

$$I_{c6} = \frac{I_{c2}}{1 + \exp(-V_1 / V_T)}$$



Gilbert cell - DC Analysis

- ❖ The two currents I_{c1} and I_{c2} are related to V_2

$$I_{c1} = \frac{I_{EE}}{1 + \exp(-V_2/V_T)} \quad I_{c2} = \frac{I_{EE}}{1 + \exp(V_2/V_T)}$$

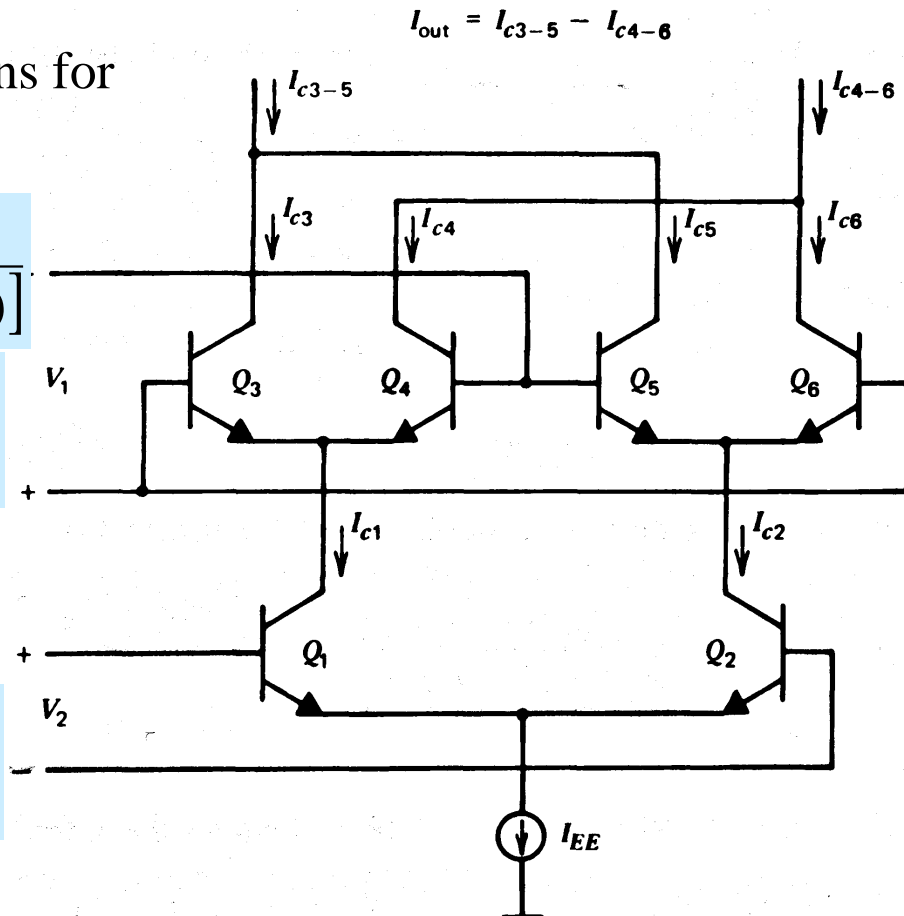
- ❖ Substituting I_{c1} and I_{c2} in expressions for
- ❖ I_{c3} , I_{c4} , I_{c5} and I_{c6} get :

$$I_{c3} = \frac{I_{EE}}{[1 + \exp(-V_1/V_T)][1 + \exp(-V_2/V_T)]}$$

$$I_{c4} = \frac{I_{EE}}{[1 + \exp(V_1/V_T)][1 + \exp(-V_2/V_T)]}$$

$$I_{c5} = \frac{I_{EE}}{[1 + \exp(V_1/V_T)][1 + \exp(V_2/V_T)]}$$

$$I_{c6} = \frac{I_{EE}}{[1 + \exp(-V_1/V_T)][1 + \exp(V_2/V_T)]}$$



Gilbert cell Applications

- ❖ The differential output current is then given by

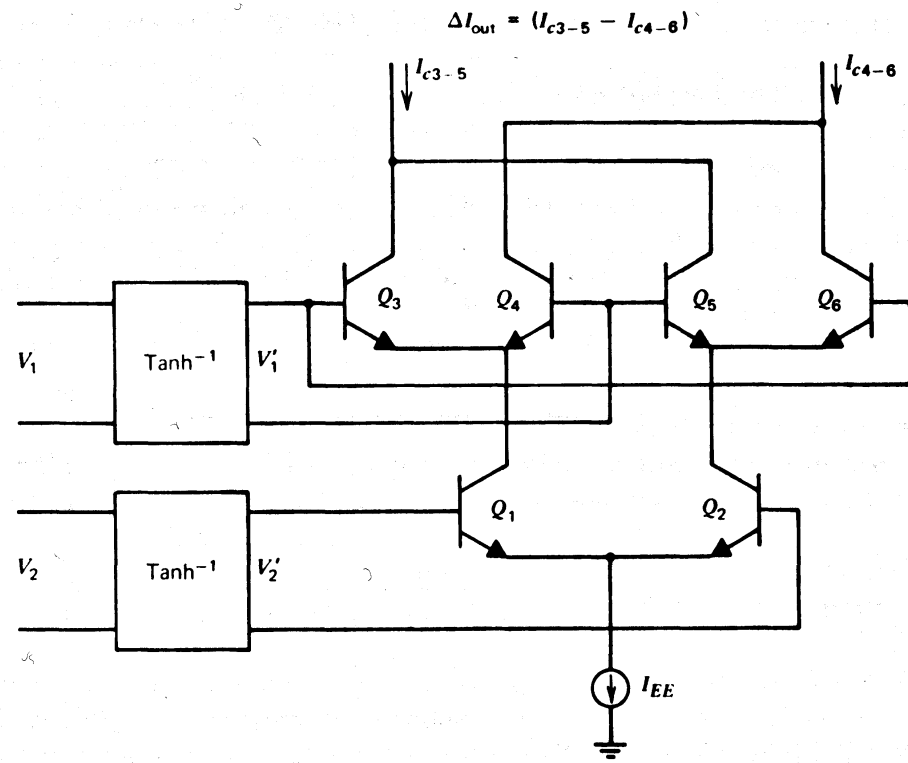
$$\begin{aligned} \Delta I &= I_{c3-5} - I_{c4-6} = I_{c3} + I_{c5} - (I_{c4} + I_{c6}) = (I_{c3} - I_{c6}) - (I_{c4} - I_{c5}) = \\ &= I_{EE} \tanh(V_1 / 2V_T) \tanh(V_2 / 2V_T) \end{aligned}$$

- ❖ The dc transfer characteristic, then, is the product of the hyperbolic tangent of the two input voltages. There are three main applications of the Gilbert cell depending on the V_1 and V_2 range:
 - ❖ (1) If $V_1 < V_T$ and $V_2 < V_T$ then: $\tanh(V_{1,2} / 2V_T) \cong V_{1,2} / 2V_T$ and it works as a multiplier
 - ❖ (2) If one of the inputs is a signal that is large compared to V_T , this effectively multiplies the applied small signal by a square wave, and acts as a modulator.
 - ❖ (3) If both inputs are large compared to V_T , and all six transistors in the circuit behave as nonsaturating switches. This is useful for the detection of phase differences between two amplitude-limited signals, as is required in phase-locked loops, and is sometimes called the phase-detector mode.

Gilbert cell as Multiplier

(1) If $V_1 < V_T$ and $V_2 < V_T$ then : $\tanh(x) = x + x^3 / 3 + \dots \cong x$

- ❖ Thus for small-amplitude signals, the circuit performs an analog multiplication. Unfortunately, the amplitudes of the input signals are often much larger than V_T
- ❖ An alternate approach is to introduce a nonlinearity that predistorts the input signals to compensate for the hyperbolic tangent transfer characteristic of the basic cell.
- ❖ The required nonlinearity is an inverse hyperbolic tangent characteristic



Pre-warping circuit - inverse hyperbolic tangent

- ❖ We assume for the time being that the circuitry within the box develops a differential output current that is linearly related to the input voltage V_i . Thus

$$I_1 = I_{o1} + K_1 V_1 \quad \text{and} \quad I_2 = I_{o1} - K_1 V_1$$

- ❖ Here I_{o1} is the dc current that flows in each output lead if V_i is equal to zero, and K_1 is the transconductance of the voltage-to-current converter

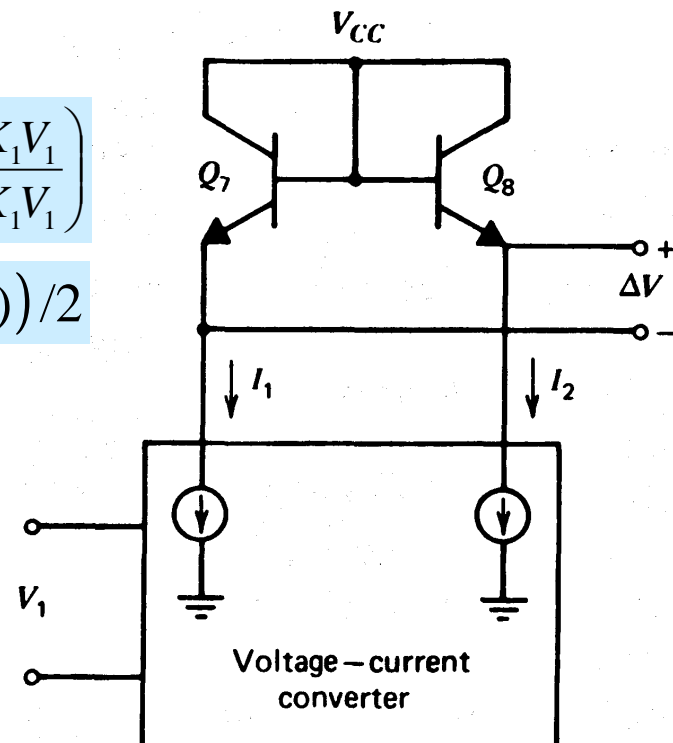
- ❖ The differential voltage developed across the two diode-connected transistors is

$$\Delta V = V_T \ln\left(\frac{I_{o1} + K_1 V_1}{I_s}\right) - V_T \ln\left(\frac{I_{o1} - K_1 V_1}{I_s}\right) = V_T \ln\left(\frac{I_{o1} + K_1 V_1}{I_{o1} - K_1 V_1}\right)$$

- ❖ Using the identity: $\tanh^{-1} x = \ln((1+x)/(1-x))/2$

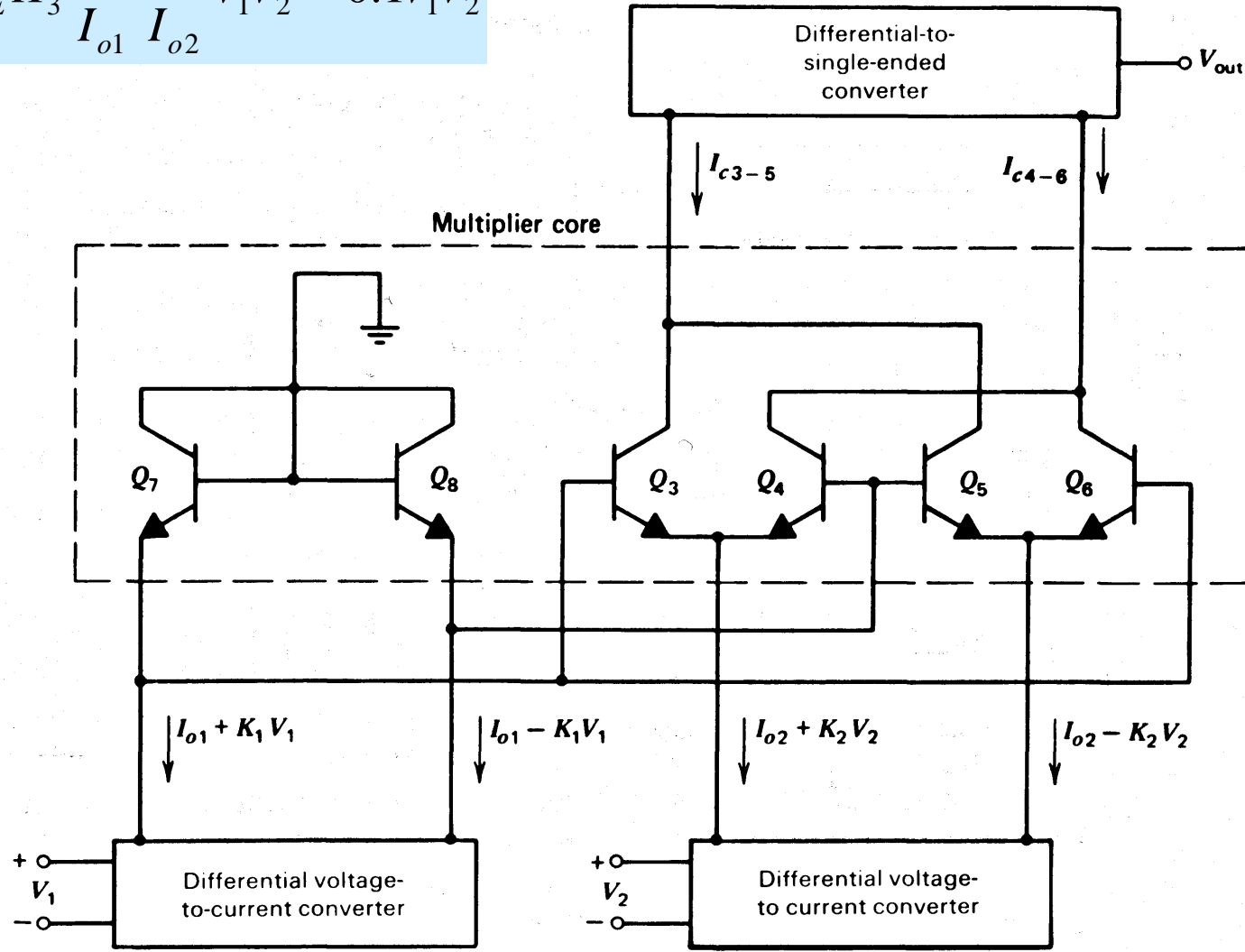
- ❖ We get
$$\Delta V = 2V_T \tanh^{-1}\left(\frac{K_1 V_1}{I_{o1}}\right)$$

- ❖ And finally
$$\Delta I = I_{EE} \left(\frac{K_1 V_1}{I_{o1}}\right) \left(\frac{K_2 V_2}{I_{o2}}\right)$$



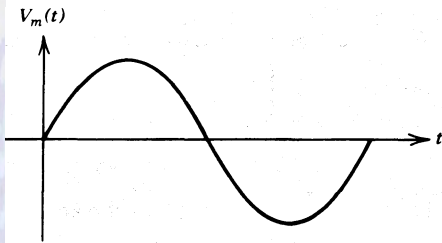
Complete Analog Multiplier

$$V_{out} = I_{EE} K_3 \frac{K_1}{I_{o1}} \frac{K_2}{I_{o2}} V_1 V_2 = 0.1 V_1 V_2$$

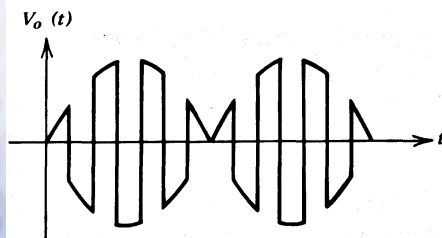
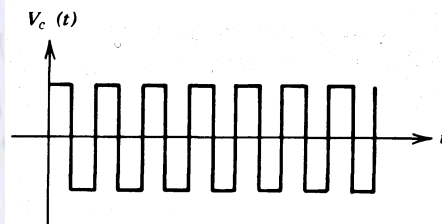


Gilbert cell as a Balanced Modulator

- ❖ In communications systems, the need frequently arises for the multiplication of a continuously varying signal by a square wave.
- ❖ This is easily accomplished with the multiplier circuit by applying a sufficiently large signal directly to the cross-coupled pair.



Large – signal modulating input



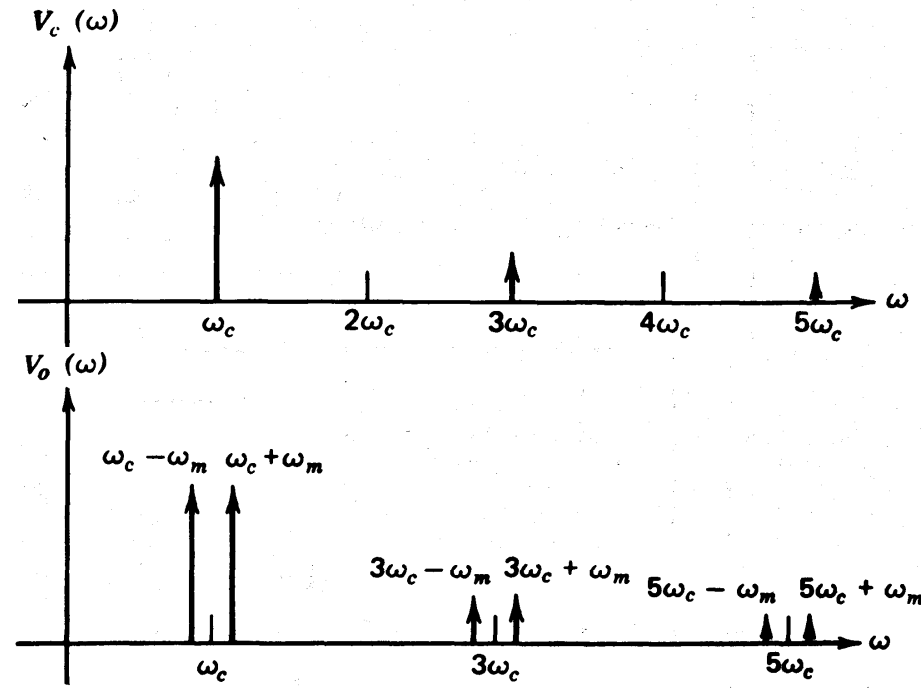
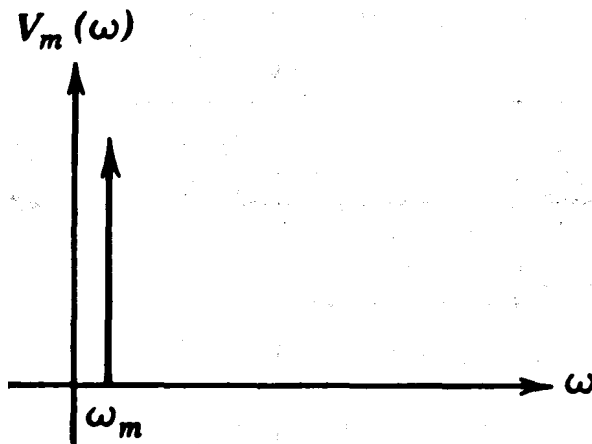
$$V_m(t) = V_m \cos \omega_m t$$

$$V_c(t) = \sum_{n=1}^{\infty} A_n \cos n\omega_c t, \text{ where } A_n = \sin\left(\frac{n\pi}{2}\right) / \frac{n\pi}{4}$$

$$\begin{aligned} V_o(t) &= K[V_c(t)V_m(t)] = K \sum_{n=1}^{\infty} A_n V_m \cos \omega_m t \cos n\omega_c t = \\ &= K \sum_{n=1}^{\infty} \frac{A_n V_m}{2} \cos(n\omega_c t - \omega_m t) \cos(n\omega_c t + \omega_m t) \end{aligned}$$

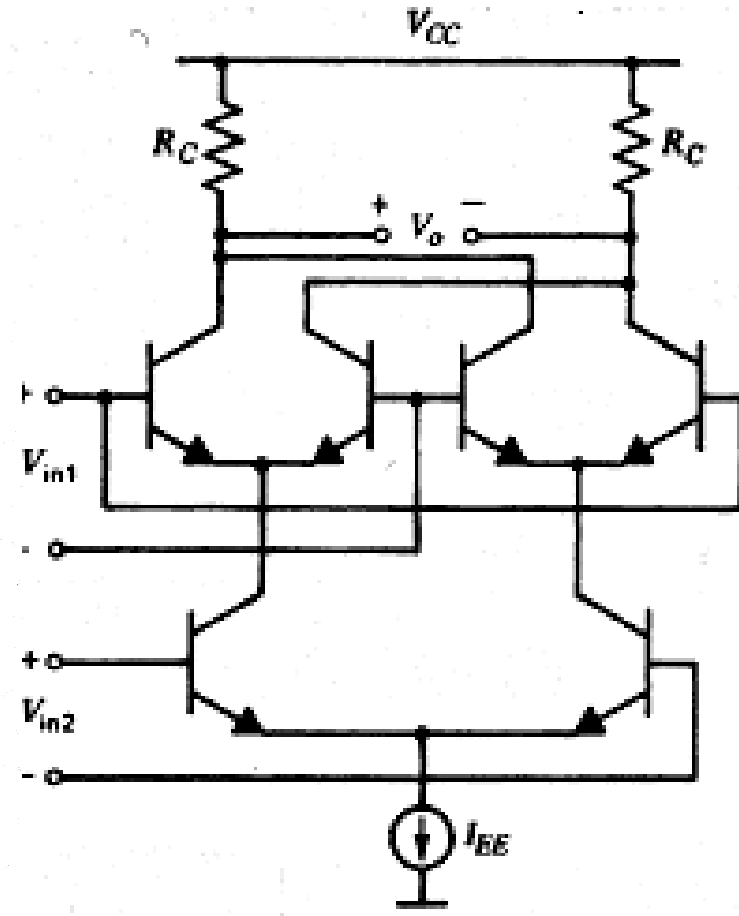
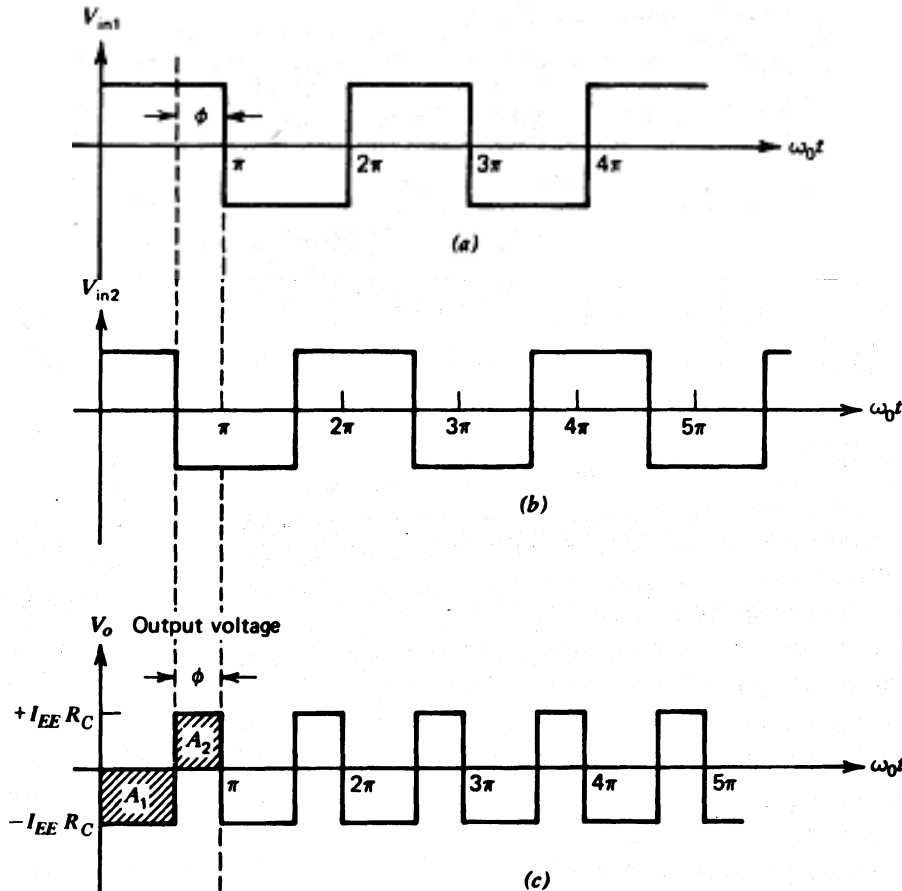
Spectra for balanced modulator

- ❖ The spectrum has components located at frequencies ω_m above and below each of the harmonics of ω_c , but no component at the carrier frequency ω_c or its harmonics. The spectrum of the input signals and the resulting output signal is shown below.
- ❖ The lack of an output component at the carrier frequency is a very useful property of balanced modulators. The signal is usually filtered following the modulation process so that only the components near ω_c are retained



Gilbert cell as a phase detector

- If unmodulated signals of identical frequency ω_0 are applied to the two inputs, the circuit behaves as a *phase detector* and produces an output whose dc component is proportional to the phase difference between the two inputs.



- ❖ The output waveform that results is shown in Fig. 10.16c and consists of a dc component and a component at twice the incoming frequency. The dc component is given by:

$$V_{average} = \frac{1}{2\pi} \int_0^{2\pi} V_o(t) d(\omega_o t) = \frac{-1}{\pi} [A_1 - A_2]$$

- ❖ where areas A_1 and A_2 are as indicated. Thus

$$V_{average} = - \left[I_{EE} R_C \frac{\pi - \phi}{\pi} - I_{EE} R_C \frac{\phi}{\pi} \right] = I_{EE} R_C \left(\frac{2\phi}{\pi} - 1 \right)$$

- ❖ If input signals are comparable to or smaller than V_T , the circuit still acts as a phase detector.
- ❖ However, the output voltage then depends both on the phase difference *and* on the *amplitude* of the two input waveforms

