OPERATING PRINCIPLE OF HYDROFOIL

- **Hydrofoil is used to produce lift in a fluid stream or water like an aerofoil.**
- **It is shaped to move smoothly through water causing the flow to be delivered which according to Newton's third law of motion exert an upward force on the foil.**
- **The turning of water causes higher pressure on the bottom and produces the pressure on the top of the foil.**
- **The pressure difference is accompanied by the velocity difference via the Bernouli's principle so that the resulting flow field about the foil has a higher average velocity on one side than the other side.**
- **Hydrofoil can be artificial such as a rudder in a boat, the diving plane or a submarine surfboard fins .**
- **Hydrofoil is used for the wing-like structure mounted on struts below the hull of the boats which lift it out of the water during forward motion in order to reduce the hull drag.**
- **When used as a lifting element , the hydrofoil beat the upward force, lift the body of the vessel , decreasing drag and increasing speed.**
- **The lifting force eventually balances with the weight of the craft reaching a point where hydrofoil no longer remains in equilibrium**
- **Wave resistance and other impending forces are eliminated as the hull is lifted**
- **Turbulence and drag act only on much smaller area of the hydrofoil and there is a remarkable increase in speed.**

LAMINAR INTERNAL FLOW

- **A laminar flow is one in which paths taken by the individual fluid particles do not cross one another.**
- **The flow is a gentle type ant there is an orderliness of the fluid particles along a well –defined path.**
- **This type of flow is called streamline or viscous flow**

Examples of this type of flow are :

(1) The flow of fluid through a capillary tube
(2) Underground flow
(3) Flow of oil in a measuring instrument

CHARACTERISTICS OF LAMINAR FLOW

(1) Fluid particles moves in a well-defined path
(2) No slip at the boundary (no movement of the fluid molecules at the boundary)
(3) Due to viscosity there is no shear between fluid layer ; this is govern by the equation .

$\tau = \mu \frac{du}{dy}$ - - - ---- -- - -- -- -- -- -- -- --- - -- -- (1)

$where\ \ \tau$ - shear stress (N/m^2)

μ - coefficient of dynamic viscosity (NS/m^2)

$\frac{du}{dy}$ - velocity gradient (S^{-1})

In laminar flow , fluid resistance which is measured by shear stress τ is related to fluid viscosity and velocity gradient in the transverse direction (x – direction) by equation 1

du= change in velocity in the flow direction (m/s)

dy= change in distance from the wall (m)

(4) The flow is rotational
(5) Due to viscous shear , there is dissipation of energy and for maintaining the flow , energy must be supplied externally .
(6) Loss of energy is proportional to first power of velocity and first power of viscosity.
(7) No mixing between different fluid layers (i.e the flow is gentle)
(8) It is characterized by small to medium scale velocity
(9) The flow remains laminar as long as $\rho uD/\mu$ is less than the critical value of Reynold's number (2,000)

REYNOLD'S NUMBER

Reynold from his experiment found that , the nature of fluid I a closed conduit (pipe) depends on the following:

(1) The pipe diameter " D" (m)
(2) Density of the fluid "ρ" (kg/m^3)
(3) Velocity of flow "v" (m/s)
(4) Viscosity of the fluid "μ " (Ns/m^2)

By combining these four variables Reynold determined a non-dimensional quantity equal to :

$$Re = \frac{\rho uD}{\mu} \quad \text{---------------------(2)}$$

In some cases , the diameter D is replaced by the characteristics length (L), then equation (2) becomes

$$Re = \frac{\rho uL}{\mu} \quad \text{----------------------(3)}$$

Also note that : the relationship between kinematic and dynamic viscosity is given as :

$$\gamma = \frac{\mu}{\rho} \quad (m^2/s) \quad \text{-- - - - - - - - -----(4)}$$

$$\frac{1}{\gamma} = \frac{\rho}{\mu} \quad \text{-- - - - - - --------(5)}$$

Substitute (5) into (6)

Re = $\frac{UL}{\gamma}$ --- - - - - - - - ----------- (6)

Conditions of flow :

Re < 2000 : Laminar flow

Between 2000 – 4000 : Transitional or Intermediate flow

Re > 4000 : Turbulent flow

Example

Air flows through a circular pipe of 1cm diameter at an average velocity of 2m/s. At room temperature , the value of coefficient of viscosity and density are : 1.983 $\times\ 10^{-5}$ kg/ms and 1774 kg/m^3 respectively . Determine the nature of flow and hence estimate the rate of flow.

Solution

(i)

D = 1cm = 1/100 = 0.01m

ρ= 1774 kg/m^3

μ= 1.983 $\times\ 10^{-5}$ kg/ms

u= 2 m/s

$Re = \frac{\rho u D}{\mu}$

Re = $\frac{1774\ \times 2\ \times 0.01}{1.983\ \times\ 10^{-5}}$

Re= 1187.49

Since Re < 2000 : The flow is laminar .

(ii)

The flow rate Q.

From the continuity equation ; $Q = Au$

A = $\frac{\pi D^2}{4}$

A = $\frac{3.142\ \times 0.01^2}{4}$

A = 7.855 $\times 10^{-5}$ m^2

Q = 7.855 $\times 10^{-5} \times 2.0$ = 1.571 $\times 10^{-4}$ m^3/s

COUTTE FLOW

This can be defined as a flow between two parallel plate in which one is at rest while the other moves with a velocity that tends to infinity .

Consider a laminar flow between the parallel flat plates located at a distance "b' apart such that the lower plate is at rest and the upper plate moves uniformly with a velocity " u " ax shown below . The force acting on the fluid elements are as shown in the figure 1

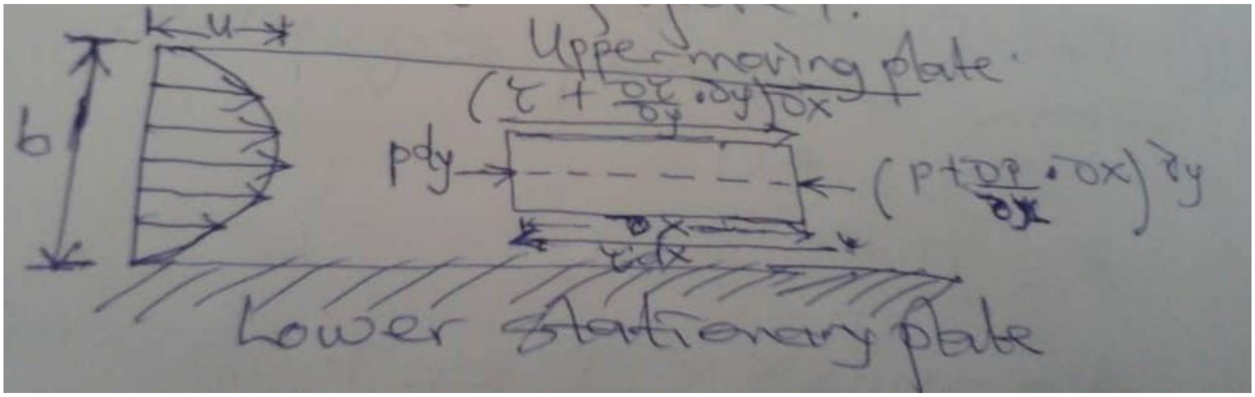

Figure 1

The following forces are in operations:

(i) **The pressure force Pdy on the left side**
(ii) **The pressure force ($P + \frac{\partial p}{\partial x}$.dx) dy on the right hand side**
(iii) **The shear force τdx on the lower surface**
(iv) **The shear force ($\tau + \frac{\partial \tau}{\partial y}$. dy) dx on the upper surface.**

Note ; The pressure gradient $\frac{\partial P}{\partial x}$ = 0

In other words , pressure gradient is constant in the x- direction . Consequently , since the flow is both steady and uniform, acceleration is <u>zero</u> . Hence the resultant force in the direction of flow is <u>zero .</u>

<u>Therefore ;</u>

$Pdy - (P + \frac{\partial p}{\partial x}.dx) dy - \tau dx + (\tau + \frac{\partial \tau}{\partial y}. dy) dx = 0$ ---------------- (1)

Pdy – pdy - $\frac{\partial p}{\partial x}$.dx dy - τdx + τdx + $\frac{\partial \tau}{\partial y}$. dy dx = 0 ………………….(2)

$- \frac{\partial p}{\partial x}. dx\,dy + \frac{\partial \tau}{\partial y}. dy\,dx = 0$ …………………..(3a)

$(-\ \frac{\partial p}{\partial x}. + \frac{\partial \tau}{\partial y}$) dy dx = 0

$-\ \frac{\partial p}{\partial x}. + \frac{\partial \tau}{\partial y}$ = 0 …………………………(3b)

The pressure gradient in the flow direction is equal to the shear across the flow.

According to Newton's law of viscosity for laminar flow: Shear stress is given as :

$\tau = \mu \frac{\partial u}{\partial y}$ …………………………..(4)

Substitute equation (4) into (3b)

$-\ \frac{\partial p}{\partial x}. + \frac{\partial\ (\mu \frac{\partial u}{\partial y})}{\partial y}$ = 0

$\frac{\partial p}{\partial x}.$ = $\frac{\partial C\ \mu \frac{\partial u}{\partial y})}{\partial y}$ …………………… (5a)

$\frac{\partial p}{\partial x}.$ = $\frac{\mu\ d^2 u}{\partial y^2}$ --------------------- (5b)

By rearranging the equation we obtained : .

$\frac{d^2 u}{\partial y^2}$.= $\frac{dp}{dx} . \frac{1}{\mu}$ ------------------------(6)

Integrating twice with respect to y to get the velocity " u"

$\frac{\partial u}{\partial y} = \frac{1}{\mu .} . \frac{\partial p}{\partial x}$. y + c_1 ……………………….(7)

$u = \frac{1}{\mu .} \frac{\partial p}{\partial x}. y^2 + c_1$y $+ c_2$ -------------------(8)

Where ci and c2 are the constants of integration and these can be found from known boundary conditions :

In the present case : the boundary conditions are : y=0, u=0, c2 = 0

$u = \frac{1}{\mu} \frac{\partial p}{\partial x}. \frac{y^2}{2}. + c_1$y -------------------------(9a)

$u = \frac{1}{2\mu} \frac{\partial p}{\partial x}. y^2. + c_1$y -----------------------(9b)

Then y = b

Where y is the distance between a plate and a particular reference point and b is the distance between the two plates .

u = U , y = b

$U = \frac{1}{2\mu}\frac{\partial p}{\partial x}.b^2. + c_1 b$

Divide both sides by b

$\frac{U}{b} = \frac{1}{2\mu}\frac{\partial p}{\partial x}.$ **b. +** c_1

$c_1 = \frac{U}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)$**. b. …………………(10)**

Substitute (10) into (9b)

$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right). y^2 + \left(\frac{U}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right). b\right) y$ **…………… (11)**

$u = \frac{Uy}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).(\text{by} - y^2)$ **………………………… (12a)**

Equation (12a) can be written as :

$u = \frac{Uy}{b} - \frac{1}{2\mu}\frac{\partial (P+\rho g z)}{\partial x}.(\text{by} - y^2)$ **---------------- (12 b)**

In case of piezometric pressure P = $P + \rho g z$**. Equation (12a) and (12b) indicate that the velocity distribution in coutte flow depends on both u and** $\frac{\partial p}{\partial x}$**.**

However, the pressure gradient can be either negative or positive and when $\frac{\partial p}{\partial x}$ $is\ zero$ **there is no pressure gradient in the direction of flow.**

Hence equation (12a) and (12b) becomes : $u = \frac{Uy}{b}$ **-------------- (13)**

Equation(13) indicates that the velocity distribution is lower . This particular case is known as a simple or plain coutte flow otherwise referred to as a simple shear flow.

In addition ,the volumetric flow rate (q) otherwise known as the discharged per unit width may be obtained as

dq = udy ------ ---------- (14a)

Integrate taking upper and lower limits as ' b" and "0"

$q = \int_0^b udy$ **-- - - - - ---------(14b)**

Substitute " u " from equation (12a) into (14b) $\left[\frac{U}{b}.\frac{b^2}{2}\right]_0$

$$q = \int_0^b \left(\frac{Uy}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x}.(\text{by} - y^2)\right)\text{dy} \quad \text{-------- ----- ---- (15)}$$

$$q = \left[\frac{U}{b}.\frac{y^2}{2}\right]_0^b - \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{by^2}{2} - \frac{y^3}{3}\right]_0^b$$

y = b, y = 0

$$q = -\left[\frac{U}{b}\cdot\frac{b^2}{2}\right] - \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{b\,.b^2}{2} - \frac{b^3}{3}\right]$$

$$q = \frac{Ub}{2} - \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{b^3}{6}\right] \quad \text{-------------------(16a)}$$

$$q = \frac{Ub}{2} - \frac{b^3}{12\mu}\frac{\partial p}{\partial x} \quad \text{……………………… (16b)}$$

SHEAR STRESS DISTRIBUTION

$$\tau = \mu\frac{\partial u}{\partial y} \quad \text{………………………… (17a)}$$

Substitute velocity " u" as defined by equation (12a) into (17a)

Recall : u = $\frac{Uy}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{by} - y^2\right)$

Hence ; $\frac{\partial u}{\partial y} = \frac{U}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{b} - \mathbf{2y}\right)$ **---------------------- (17b)**

$$\tau = \mu\left(\frac{U}{b} - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{b} - \mathbf{2y}\right)\right) \quad \text{------------------------------- (18a)}$$

$$\tau = \frac{\mu U}{b} - \frac{\mu}{2\mu}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{b} - \mathbf{2y}\right) \quad \text{----------------------------------- (18b)}$$

$$\tau = \frac{\mu U}{b} - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{b} - \mathbf{2y}\right) \quad \text{----------------------- --------------(19)}$$

In summary : For coutte flow :

- ✓ **The velocity is defined by equation (12a) or (12b)**
- ✓ **The flowrate is defined by equation (16a) and (16b)**
- ✓ **Shear stress is defined by (18b)**

However , when both plates are at rest ; the velocity , flow rate and shear stress can be obtained from equations :

Both plates at rest put " U " = 0

Substitute " U " = 0 in (12a)

u = $-\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).\left(\mathbf{by} - y^2\right)$ **……………………….. (19)**

Flow rate : Put " U " = 0 into equation (16b)

$$q = -\frac{b^3}{12\mu}\frac{\partial p}{\partial x} \quad \text{---------------------------- (20)}$$

For shear stress: substitute " U " = 0 into equation (18b)

$$\tau = -\frac{\mu}{2}\left(\frac{\partial p}{\partial x}\right).(\mathbf{b} - 2\mathbf{y}) \quad \text{--------------------- -------(21)}$$

EXAMPLE 1

Determine the direction and amount of flow per unit metre between two parallel plates is moving relative to the other with a velocity of 3 m/s in the negative direction . If the pressure gradient $\frac{\partial p}{\partial x}$ = -100 $\times$ 10^6 N/m^3 and coefficient of dynamic viscosity μ = 0.4 Poise .

SOLUTION

Velocity , u = - 3m/s

Coefficient of dynamic viscosity μ = 0.4 Poise

Pressure gradient $\frac{\partial p}{\partial x}$ = -100 $\times$ 10^6 N/m^3

Convert from Poise to SI unit :

10 Poise = 1.0 Ns/m^2

0.4 Poise = $\frac{0.4 \times 1.0}{10}$ Ns/m^2 = 0.04 Ns/m^2

viscosity μ = 0.04 Ns/m^2

Distance of separation " b" = 1mm = 0.001m

$$q = \frac{Ub}{2} - \frac{b^3}{12\mu}\frac{\partial p}{\partial x}$$

$$q = \frac{-3 \times 0.001}{2} - \frac{(1.0 \times 10^{-3})^3}{12 \times 0.04}(-100 \times 10^6)$$

$$q = 0.21\, m^3/s$$

The direction is positive to that of moving plate.

EXAMPLE 2

Two parallel plates kept at 100mm apart have laminar flow of oil between them with maximum velocity of 1.5 m/s. Calculate : (a) discharge per metre (b) shear stress at the plate (c) pressure difference between the two points 20m apart (d) velocity gradient at the plates (e) velocity distribution at 20mm from the plate (Assume oil viscosity = 24.5 Poise)

SOLUTION

Distance between the two plates : "b" = 100 mm = 0.1 m

Maximum flow velocity " u_{max}" = 1.5 m/s

Oil viscosity " μ " = 24.5 Poise

Conversion from Poise to SI unit ;

10 Poise = 1.0 Ns?m^2

24.5 Poise = $\frac{24.5 \times 1.0}{10}$

μ**= 2.45 Ns/m^2**

(a) Discharge per unit width " q "

$q = \overline{u} \times b$

Note that for parallel plate

$\overline{u} = \frac{2}{3} u_{max}$

For a circular pipe **;**

$\overline{u} = \frac{1}{2} u_{max}$

Hence :

$q = \frac{2}{3} u_{max} .b$

$q = \frac{2}{3} \times 1.5 \times 0.1$

$q =$ **0.1 m^3/s/m**

(b) Stress at the plate :

$$q = -\frac{b^3}{12\mu} \frac{\partial p}{\partial x}$$

$$0.1 = -\frac{0.1^3}{12 \times 2.45} \frac{\partial p}{\partial x}$$

$\frac{\partial p}{\partial x}$ **= - 2940 N/m^2/m**

Shear stress

$$\tau = -\frac{1}{2} \left(\frac{\partial p}{\partial x}\right).(\mathbf{b} - \mathbf{2y})$$

@ y = 0

$$\tau = -\frac{1}{2}(-2940).(0.1 - 2(0))$$

τ = 147 N/m^2

(c) Pressure difference between the two plates 20m apart ;

$-\frac{\partial p}{\partial x}$ = 2940

$-dp = 2940\ dx$

$\int_{p1}^{p2} - dp$ = $\int_{x1}^{x2} 2940$ dx

$-\{p2 - p1\}$ = $2940\ [X2 - X1]$ =

$\{p1 - p2\}$ = $2940\ [X2 - X1]$

ΔP = 2940 (20 -0)

ΔP = 58,800 N/m^2

ΔP = 58.8 kN/m^2

(c) Velocity gradient ; $\frac{\partial u}{\partial y}$

$\tau_o = \mu \left[\frac{\partial u}{\partial y}\right]$ @ y=0

$\frac{\partial u}{\partial y} = \frac{\tau_o}{\mu}$

$\frac{\partial u}{\partial y} = \frac{147}{2.45}$

$\frac{\partial u}{\partial y}$ = 60 s^{-1}

(e) Velocity distribution at 20mm from the plate . y= 20mm = 0.02 m

u = $-\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right).(\mathrm{by} - y^2)$

u = $-\frac{1}{2\times 2.45}(-2940).(0.1 \times 0.02 - 0.02^2)$

u = $0.96\ m/s$

ASSIGNMENT

SUBMIT TO THE Email : rominiyiol@abuad.edu.ng / engromslawani@yahoo.com

DEADLINE : 10 TH APRIL , 2020 ON OR BEFORE 12 MID NIGHT

NOTE : Any Assignment submitted after this given ultimatum will not be marked and graded. (STAY SAFE)

QUESTION 1

(a)

(i) Give three conditions for coutte flow
(ii) State four conditions that can be used to determine the nature of flow.
(iii) In tabular form differentiate between aerofoil and hydrofoil.

(b)

A liquid of 0.9 Centipoise is filled between two horizontal plate of 10mm apart. If the upper plate moves at I m/s relative to the lower plate which is stationary and the pressure difference between the two sections 60 mmm apart is 60 kN/m^2 . Compute : (i) velocity distribution (ii) discharge per unit width

(iii) shear stress at the upper plate

QUESTION 2

Laminar flow of a liquid in fig2, whose viscosity is 0.9 Ns/m^2 and density of 1260 kg/m^3 occurs between a pair of parallel plates of extensive width inclined at an angle 45^0 to the horizontal. The upper plate moved with a velocity of 1.5 m/s relative to the lower plate in a direction opposite to the fluid flow . Pressure gauges mounted at two points 1m vertically apart on the upper plate records a pressure of 250 kN /m^2 and 80 kN /m^2 respectively.

Determine (a) Velocity and shear stress distribution between the plates

(b) maximum flow velocity

(c) Shear stress on the upper plate.

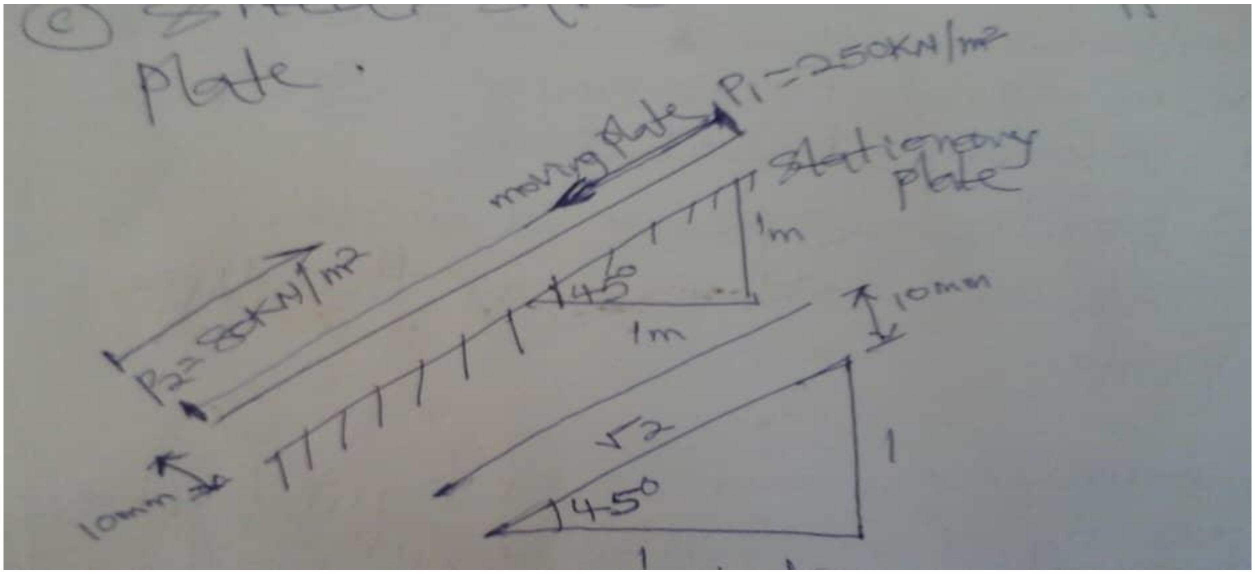

FIG2

Ans : (a) (i) Velocity distribution ; $u = 566.4y - 7.164 \times 10^{-4}\, y^2$

(ii) Shear stress distribution $\tau = .509.76 - 1.289 \times 10^5 y$

(b) u_{max} (maximum velocity) = 1.12 m/s , $\tau\ @\, y = 3.9530 \times 10^{-3}\, m = 0.218\, N/m^2$

(c) Shear stress at the upper plate : $\tau\ (upper\ plate)$**= - 0.78 N/m^2**