Methods of Integration
1. Integration of linear fns
2. Integration by Algebraic substitution
3. Integrals of the form $\int \frac{g'(x)}{g(x)}$ and $\int g(x)g'(x)dx$
4. Integration by partial fractions
5. Integration of rational algebraic fractions ***
6. Integration by parts
7. Integration of trigonometric fns.

1. <u>Integration of linear fns</u> (Examples).

$$\int(2x^3-4x^2-7x+1)dx$$
$$=\frac{2x^4}{4}-\frac{4x^3}{3}-\frac{7x^2}{2}+x+C$$
$$=\frac{x^4}{2}-\frac{4x^3}{3}-\frac{7x^2}{2}+x+c$$

$$\int 3\sqrt{x^5}\,dx$$
$$=3\int(x^5)^{1/2}dx$$
$$=3\int x^{5/2}dx$$
$$=3\left[\frac{x^{7/2}}{7/2}\right]+C$$
$$=3\left[x^{7/2}\times\frac{2}{7}\right]=\frac{6x^{7/2}}{7}+C$$

- Write standard integrals here

2) <u>Integration by algebraic sub</u>

1) $$\int\frac{dx}{(4-3x)^4}=\int(4-3x)^{-4}dx$$

Let $u=4-3x$

$\frac{du}{dx}=-3 \Rightarrow du=-3dx$, $dx=\frac{-du}{3}$

OR $\int u^{-4}\cdot\frac{-du}{3}$
$=-\frac{1}{3}\int u^{-4}du$

$$\int\frac{-du/3}{u^4}\Rightarrow\int-\frac{du}{3}\times\frac{1}{u^4}$$
$$=-\frac{1}{3}\int\frac{du}{u^4}=-\frac{1}{3}\left[u^{-4}\right]du$$
$$=-\frac{1}{3}\left[\frac{u^{-3}}{-3}\right]+c$$
$$=\frac{1}{9}\left[u^{-3}\right]+c$$
$$=\frac{1}{9}\left[4-3x\right]^{-3}+c$$

(2)

$$\int\cos(5x+3)dx$$

Let $u=5x+3$, $\frac{du}{dx}=5$

$dx=du/5$

$$\int\cos u\,du/5=\frac{1}{5}\int\cos u\,du$$
$$=\frac{1}{5}\left[\sin u\right]+C$$
$$=\frac{1}{5}\sin(5x+3)+C$$

Try

$$\int(2x-5)^7dx$$
$$=\frac{1}{16}(2x-5)^8+C$$

$\Rightarrow$ Let $u=2x-5$
$du=2dx$
$dx=du/2$

$\int u^7\frac{du}{2}$
$=\frac{1}{2}\left[\frac{u^8}{8}\right]$

$=\frac{1}{16}(2$

(3)

$$\int \frac{dx}{\sqrt{x+2}}$$

Let $u = \sqrt{x+2}$, we can write

$u^2 = x+2 \Rightarrow x = u^2-2$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

we have

$$\int \frac{2u\,du}{u} = \int 2\,du$$

$= 2u + C$

$= 2\sqrt{x+2} + C$

OR

$u = x+2$, $\frac{du}{dx} = 1$

$du = dx$

$$\int \frac{du}{u^{1/2}} = \int u^{-1/2}du$$

$= \frac{u^{1/2}}{1/2} = 2u^{1/2}$

$= 2(x+2)^{1/2} + C$

(4)

$$\int x\sqrt{2x+1}\,dx$$

Let $u = \sqrt{2x+1}$

$u^2 = 2x+1$

$x = \frac{u^2-1}{2}$

$\frac{dx}{du} = \frac{2u}{2} = u \Rightarrow dx = u\,du$

we have

$$\int \frac{u^2-1}{2} \cdot u \cdot u\,du$$

$$\Rightarrow \int \frac{u^2(u^2-1)}{2}du$$

$$\int \frac{u^4-u^2}{2} = \frac{1}{2}\left[\frac{u^5}{5} - \frac{u^3}{3}\right]$$

$\left(\sqrt{2x+1}\right)^5 - \left(\sqrt{2x+1}\right)^3 + C$

$u = 2x+1$, $\frac{du}{dx} = 2$

$dx = du/2$

$x = \frac{u-1}{2}$

$$\int \frac{u-1}{2} \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{1}{4}\int u^{1/2}(u-1)\,du$$

$$\frac{1}{4}\int (u^{3/2} - u^{1/2})\,du$$

$\frac{1}{4}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]$

$\frac{u^{5/2}}{10} - \frac{u^{3/2}}{6}$

$$\int \frac{\theta^4 - \theta^2}{\theta}d\theta$$ **

(5)

$$\int x\sqrt{9+x^2}\,dx$$

$u = \sqrt{9+x^2}$

$u^2 = 9+x^2$

$u^2 - 9 = x^2$

$x = (u^2-9)^{1/2}$

$\frac{dx}{du} = \frac{1}{2}(u^2-9)^{-1/2} \cdot 2u$

$\frac{dx}{du} = \frac{1}{2} \cdot \frac{1}{(u^2-9)^{1/2}} \cdot 2u$

$\frac{dx}{du} = \frac{u}{(u^2-9)^{1/2}}$

$u\,du = (u^2-9)^{1/2}dx$

$dx = \frac{u\,du}{(u^2-9)^{1/2}}$

we have

$$\int (u^2-9)^{1/2} \cdot u \cdot \frac{u\,du}{(u^2-9)^{1/2}}$$

$= \int u^2 du = \frac{u^3}{3} + C$

$= \frac{\left(\sqrt{9+x^2}\right)^3}{3} + C$

OR $u = 9+x^2$

$x = (u-9)^{1/2}$

$\frac{dx}{du} = \frac{1}{2}(u-9)^{-1/2}$

$dx = \frac{1}{2(u-9)^{1/2}}$

$dx = \frac{du}{2(u-9)^{1/2}}$

$\int (u-9)^{1/2} \cdot u^{1/2} \cdot \frac{du}{2(u}$

$\int u^{1/2}/2 = \frac{1}{2}$

⑥

$\int 2e^{6x-1}\,dx$

~~Let~~ we can write $2\int e^{6x-1}\,dx$

Let $u = 6x-1$

$\frac{du}{dx} = 6 \Rightarrow dx = \frac{du}{6}$

$= 2\int e^{u}\cdot\frac{du}{6} = \frac{1}{3}\int e^{u}\,du$

$= \frac{1}{3}e^{u} + C$

$= \frac{1}{3}e^{6x-1} + C$.

⑦

$\int 3x(4x^2+3)^5\,dx$

Let $u = 4x^2+3$

$\cancel{\frac{du}{dx}}\ x^2 = \frac{u-3}{4}$

$x = \left(\frac{u-3}{4}\right)^{1/2}$

$x = \frac{(u-3)^{1/2}}{2}$

$\frac{dx}{du} = \frac{1}{2}(u-3)^{-1/2} \times \frac{1}{2}$

$\frac{dx}{du} = \frac{1}{4}(u-3)^{-1/2} = \frac{1}{4(u-3)^{1/2}}$

$3\int \frac{(u-3)^{1/2}}{2}\cdot u^5\cdot\frac{du}{4(u-3)^{1/2}}$

$\frac{3}{8}\int u^5\,du$

$= \frac{3}{8}\frac{u^6}{6} + C$

$= \frac{u^6}{16} + C = \frac{(4x^2+3)^6}{16} + C$

③

Ass

① $\int \frac{2x}{\sqrt{4x^2-1}}\,dx = \frac{1}{2}\sqrt{4x^2-1} + C$

② $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \frac{(\sin^{-1}x)^2}{2} + C$ $\int g(x)g'(x)$

③ $\int (\tan x)^6\sec^2 x\,dx \Rightarrow$ $u = \tan x$, $du = \sec^2 x\,dx$, $u^6\,du = u^7/7$

Ans $\Rightarrow \frac{(\tan x)^7}{7} + C$

b) $\int a\,dx = ax + C$ c) $\int dx = x + C$

Some special integrals.

	$f(x)$	$\int f(x)$
1.	x^n	$\frac{x^{n+1}}{n+1} + C$ except when $n=-1$
2.	$\frac{1}{x} \Rightarrow \int\frac{dx}{x}$	$\log_e x$ or $\ln x$
3.	e^{ax}	$\frac{1}{a}e^{ax} + C$
4.	$\int e^{x}$	$e^{x} + C$.
5.	$\int \sin x$	$-\cos x + C$
6.	$\cos x$	$\sin x + C$
*7.	$\sec^2 x$	$\tan x + C$
8.	$\tan x$	$\ln\sec x$ or $-\ln\cos x + C$
9.	$\int\frac{dx}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + C$
10.	$\int\frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a} + C$
11	$\int\frac{-dx}{\sqrt{a^2-x^2}}$	$\cos^{-1}\frac{x}{a} + C$
*12	$\int \operatorname{cosec}^2 x\,dx$	$-\cot x + C$