

## Rank of a matrix

Defn: Rank of a matrix  $A$  is the order of the largest square matrix or sub matrix of  $A$  whose determinant is not equal to zero

### Steps Involved

1. Consider a matrix (strictly  $n \times n$ )
2. Find the det. of  $A$
3. If  $\det(A) = 0$ , proceed to the largest square sub matrix of  $A$
4. Find the det of  $A^{\text{sub}}$
5. If  $\det(A^{\text{sub}}) \neq 0$ , then the rank of  $A$  is that of its sub matrix.

### Example

① Find the rank of

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 0 & 5 \\ 5 & 1 & 7 \end{pmatrix}$$

Soln.

$$|A| = 3(0-5) - 1(14-25) + 2(2-5)$$

$$|A| = 3(-5) - 1(-11) + 2(-3)$$

$$|A| = -15 + 11 - 6 = 0 \text{ Hence,}$$

We need to consider a sub matrix of  
A

$$A^{\text{sub}} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$$

$$|A^{\text{sub}}| = 0 - 2 = -2 \neq 0$$

Hence, the rank of A is 2.

2- Find the Rank of

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$$

$$|A| = 1(1-9) - 2(2-0) + 0$$

$$|A| = -8 - 4 = -12 \neq 0$$

Hence, the rank of A is 3.