TURBULENT FLOW

A turbulent flow is that flow in which fluid particles move in a zig-zag manner. turbulent flow occurs in a pipe when Reynolds number is greater than 4000 (Re>4000) fluid motion becomes irregular and chaotic. Hence there is complete mixing of fluid particles due to collision of fluid mases with one another.

The following are examples of turbulent flow:

(1) High velocity flow in a conduit of large size

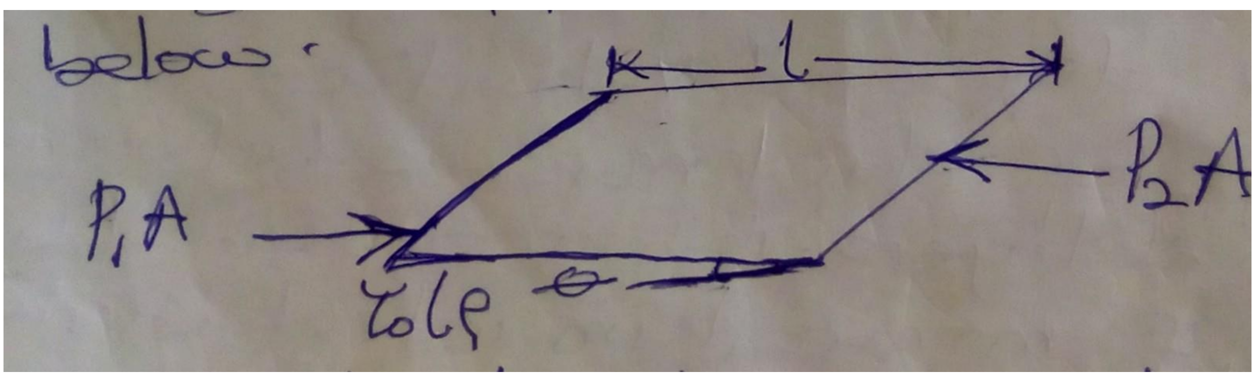
(2) Turbulent flow conditions are far more likely than laminar flow in engineering situations.

CHARACTERISTICS OF TURBULENT FLOW

(1) High velocity flow of liquid in a conduit
(2) As fluid mases in adjacent layer have different velocities, interchange of fluid mases between interchanged of fluid mases between the adjacent layers is accompanied by transfer of momentum which carries additional shear stress of high magnitude between the adjacent layers
(3) Flow is turbulent when Re>4000.
(4) Turbulent flow is also characterized by random haphazard movement of fluid particles.
(5) Velocity distribution in turbulent flow is more uniform than in laminar flow.
(6) The velocity gradient near the boundary is quite large resulting into more shear.
(7) Random orientation of fluid particles in turbulent flow give rise to additional stress called the Reynold`s stress
(8) Formation of eddies ,mixing and curing of path-lines in turbulent flow results in much greater frictional loses for the same rate of discharge viscosity or pipe sizes.

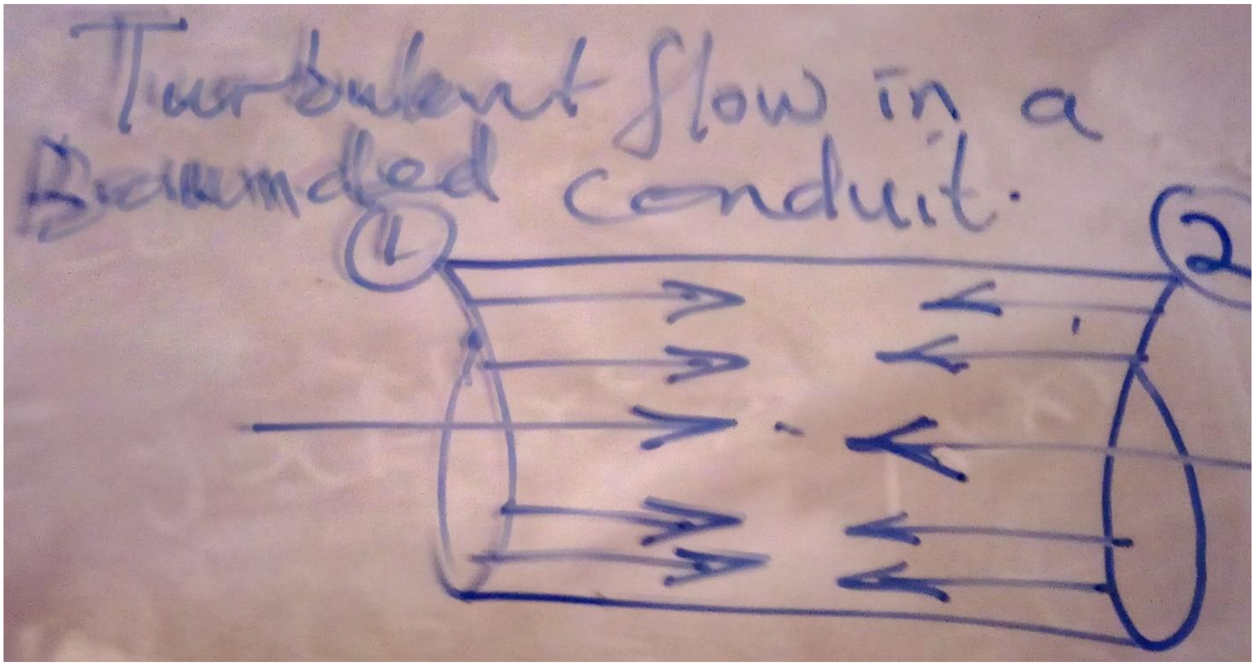
INCOMPLETE STEADY-UNIFORM BUT TURBULENT FLOW IN HORIZONTAL PIPE

Consider a small element of fluid in a horizontal pipe as shown in the figure below:



Turbulent flow in a bounded conduit. The flow is assumed to be uniform and bsteady so that fluid acceleration inn the direction of flow is zero. P1 and P2 ate the static pressure in position 1 and 2. Hence, applying the momentum equation to the fluid element in the flow direction yields.

$$P1A - P2A - \tau_0\, lp + w\sin\theta = 0 \quad \text{------------------------------(1)}$$



$P1$ = the pressure intensity at section 1

$P2$ = the pressure intensity at section 2

l= length of the pipe

$D =$ pipe diameter between section 1 and 2

$f^1 =$ Non-dimensional factor (whose value depends upon the material and nature of the pipe surface

h_f=head loss due to friction

The propelling force in the fluid flowing between the two sections is given as :

$f = (P1 - P2)$A ------------------------------------(2)

where A is the area of cross section of the pipe.

The frictional resistance force $f = f^1$plv^2 -------------------------(3)

Where p is the wetted perimeter and v is the average flow velocity. Under equilibrium condition equation 2 must be equal to equation 3 . That is the propelling force is equal to the frictional resistance force

$(P1 - P2)$A = f^1plv^2 -------------------------------(4)

By dividing both sides by the weight density w

$\frac{(P1-P2)A}{w} = \frac{f^1 plv^2}{w}$ ---------------------------------------(5)

$\frac{(P1-P2)}{w} = \frac{f^1}{w}\frac{P}{A}lv^2$ ------------------------------------(6)

$h_f = \frac{P1-P2}{w} = \frac{\Delta P}{w}$ --(7)

$Substitute\ equation$ 7 into equation 6

$h_f = \frac{f^1}{w}\frac{P}{A}lv^2$ --(8a)

$h_f = \frac{2gf^1}{w}\frac{P}{A}\frac{lv^2}{2g}$ -- (8b)

$\frac{A}{p}$ = m , Hence $\frac{p}{A} = \frac{1}{m}$ ---------------------------------- (8c)

Substitute equation 8c into 8b

$h_f = \frac{2gf^1}{w}\frac{1}{m}\frac{lv^2}{2g}$

$h_f = \frac{2gf^1}{w}\frac{l}{m}\frac{v^2}{2g}$ -- (9)

$The\ ratio$ $\frac{A}{p}$ is called the hydraulic depth and denoted by m.

The term $\frac{l}{m}\frac{v^2}{2g}$ has a dimension of h_f, thus $\frac{2gf^1}{w}$ is a non – dimensional quantity and can be replaced by a constant F.

$Therefore, substitute$ $\frac{2gf^1}{w} = F$ into equation 9

$h_f = F\frac{l}{m}\frac{v^2}{2g}$ -------------------------(10)

$In\ case\ of\ a\ circular\ pipe$, the hydraulic mean depth $m = \frac{A}{p} = \frac{\frac{\pi D^2}{4}}{\pi D}$

$m = \frac{D}{4}$ ----------------------------- (11)

$Substitute$ equation 11 into 10

$h_f = 4F\frac{l}{D}\frac{v^2}{2g}$ ----------------------- (12)

$The\ factor\ F$ is known as Darcy coefficient of friction and equation 12 is Darcy – Weisbach equation and holds for all types of flow provided a proper value of F is chosen . Sometimes equation 12 can be re-written as :

$h_f = f^1\frac{l}{D}\frac{v^2}{2g}$ ----------------------- (13) where f^1 = 4F

$Combining\ equation$ 7 and 12

$\frac{P1-P2}{w} = 4F\frac{l}{D}\frac{v^2}{2g}$ -----------------------------(14)

$P1 - P2 = 4F\frac{l}{D}\frac{v^2 w}{2g}$ --------------------------(15)

Expression for coefficient of friction in terms of stress

$(P1 - P2)A$ **= Force due to shear stress**

τ_0**= Shear stress at the pipe wall**

$(P1 - P2)A = \tau_0 \times surface\ area\ of\ the\ pipe\ wall$ --------------------(16a)

$(P1 - P2)A = \tau_0 \pi D l$ -----------------------(16 b)

$(P1 - P2)\frac{\pi D^2}{4} = \tau_0 \pi D l$ ---------------------------(17)

$Divide\ both\ sides\ by\ \pi D$

$(P1 - P2)\frac{D}{4} = \tau_0 l$ ------------------------------------ (18)

$(P1 - P2) = \frac{4\tau_o l}{D}$ ------------------------------------ (19)

$Substituting$ **equation 19 into 15**

$\frac{4\tau_o l}{D} = 4F\ \frac{l}{D}\ \frac{v^2\ w}{2g}$ - ----- -------- --- -------------- (20)

$\tau_o l = \frac{4Flv^2w}{2g}$ --------------------------------------(21)

$\tau_o = \frac{4Flv^2w}{2g}/4l$ ------------------------------ (22)

$\tau_o = \frac{fv^2w}{2g}$ ------------------------------ (23)

$Recall\ :\ w = \ \rho g$ -------------------------- (24)

$Substitute$ **24 into 23**

$\tau_o = \frac{fv^2\rho}{2}$ ----------------------------------- (25)

$f = \frac{2\tau_o}{v^2\rho}$ -- (26)

HEAD LOSS (PRESSURE DROP)

Head loss inn a pipe refers to the energy loss or pressure drop due to friction or pipe orientation. Head loss could be major or minor causes. Major loss could be caused by friction , leakages and impurities while minor loss could be as a result of pipe orientation .

Head loss could be used to determine the optimum efficiency of the piping system. It can be expressed in pressure terms of feet (ft) or the equivalent metric dimension of the fluid flowing .

EXAMPLE 1

Calculate the loss of head due to friction and the power required to maintain the flow in an horizontal circular pipe 40mm diameter whose length is 750m when water flow atb the rate of : (a) 4.0 Litre/min (b) 30 Lit/min. Assuming that the pipe has a roughness of 0.0008m , coefficient of dynamic viscosity μ = 1.14 $\times\ 10^{-3}$ Ns/m^2 .

SOLUTION

Pipe diameter D = 40 mm = 0.04m

Length L = 750m

Flow rate (Q) = 4.0 Lit/min

$\frac{4.0 \times i}{1000 \times 60}$ = 6.67 $\times\ 10^{-5}\ m^3/s$

Area (A) = $\frac{\pi D^2}{4}$

$\frac{3.142 \times 0.04^2}{4}$ = 0.00126 m^2 = 1.26 $\times\ 10^{-3}$

$Q = AV$

$V = \frac{Q}{A} = \frac{6.67\ C10^{-5}}{1.26 \times 10^{-3}}$

$V = 0.053$ m/s

Re = $\frac{\rho VD}{\mu} = \frac{1000 \times 0.053\ \times 0.04}{1.14 \times 10^{-3}}$

Re = 1859.65

Re = 1860

Since Re < 2000 : the flow is hence laminar .

For laminar flow , head loss can be calculated from Pouseiulle equation :

$P1 - P2 = \frac{32\ \mu uL}{D^2}$; $u = v$

$P1 - P2 = \frac{128\ \mu LQ}{\pi D^4}$

$$P1 - P2 = \frac{32 \times 1.14 \times 10^{-3}\ \times 0.053 \times\times 750}{0.04^2}$$

$P1 - P2$ = 907. 64 N/m^2

Recall : Head loss $h_f = \frac{P1 - P2}{w}$

$h_f = \frac{P1 - P2}{\rho g}$

h_f = $\frac{907.64}{1000\ \times 9.81}$

h_f = 0.093 m of water

= 93 mm of water.

Alternatively:

h_f = $\frac{4Flv^2}{2gD}$ Darcy's equation

Since the flow is laminar

$F = \frac{16}{Re}$

$F = \frac{16}{1859.65}$

$F =$ 0.0086

h_f = $\frac{4\ \times 0.0086\ \times 750\ \ \times 0.053^2}{2\ \times 9.81\ \times\times 0.04}$

h_f = 0.0923 m of water

= 92.3 mm of water

Power required to maintain the flow;

$p =\ \rho g h_{fQ}$

$p = 1000\ \times 9.81\ \times 0.0823\ \times 6,67\ \times\ 10^{-5}$

$p =\ 0.0604\, Watts$

CASE 2

Q = 30 Lit/min

Q = $\frac{30}{1000\ \times 60}$

5.0 $\times\ 10^{-4}$ m^3/s

A= 0,00126 m^2

V = $\frac{Q}{A}$ = $\frac{5.0 \times 10^{-4}}{0{,}00126}$

V = 0.4 m/s

Re = $\frac{\rho VD}{\mu}$ = $\frac{1000\times 0.4\ \times 0.04}{1.14 \times 10^{-3}}$

Re = 14,035

Re > 4000; Hence the flowis turbulent.

RR $= \frac{K}{d}$ **where k is the absolute roughness , d is the diameter of the pipe and RR is the relative roughness**

$= \frac{0.0008}{0.04}$ **= 0.002**

From the graph F = 0,008

An allowance of ± 0.001

0.007 – 0.009

$h_f = \frac{4Flv^2}{2gD} = \frac{4 \times 0.008 \times 750 \times 0.4^2}{2 \times 9.81 \times 0.04}$

$h_f =$ **4.89m of water**

Power required :

$p = \rho g h_f Q$

P = 1000 $\times 9.81 \times 4.89 \times 5.0 \times 10^{-4}$

P = 23.99 Watts $\approx 24\ Watts$

For Turbulent flow F $= \frac{0.0791}{Re^{0.25}}$ **note**

EXAMPLE 2

The water level in an overhead tank for a mansion's water supply system as shown in figure 3 below is 25m high . If the pipe diameter is 14cm and the discharge is 80 Lit/min over a length of 800m. Compute (a) Total head loss (b) Pressure change (c) Pumping power required. (Hint : Coefficient of dynamic viscosity = 1.14 $\times 10^{-3}$ **and the absolute roughness = 1.5** $\times 10^{-4}$ **)**

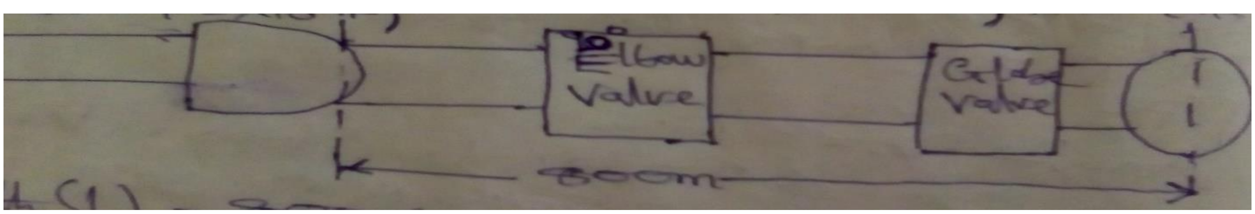


FIGURE 3

SOLUTION

Length L = 800 m

Pipe diameter D = 14 cm = 0,14 m

Flow rate Q = 80 Lit/min

Q = $\frac{80}{1000 \times 60}$

= 1.33 $\times\ 10^{-3}$ n^3?s

Area A = $\frac{\pi D^2}{4}$ = $\frac{3.142 \times 0.14^{\ 2}}{4}$ = 0.0154 m^2

From continuity equation " Q = AV

V = $\frac{Q}{A}$

V= $\frac{1.33 \times 10^{-3}}{0.0154}$

V = 0.0844 m/s

Re = $\frac{\rho V D}{\mu}$ = $\frac{1000 \times 0.0844 \times 0.14}{1.14 \times 10^{-3}}$

Re = 10,365

The flow is turbulent since Re ? 4000.

RR= $\frac{k}{D}$ = $\frac{1.5 \times 10^{\ -4}}{0.14}$ = 1.07 $\times\ 10^{-3}$

1.0 $\times\ 10^4$ against 0.00107 from the graph ; f = 0.008

Hence: Head loss h_f = $\frac{4Flv^2}{2gD} = \frac{4 \times 0.008 \times 800 \times 0.0844^2}{2 \times 9.81 \times 0.14}$

h_f = 0.066 m

The total head loss = minor loss + major loss

h_{total} = $h_m + h_f$

$h_m = \frac{kv^2}{2g}$

k = Globe vave + 90^0

From the table : k = 10 + 0.9 = 10.9 m

h_m = $\frac{10.9 \times 0.0844^2}{2 \times 9.81}$

h_m = 3.96 $\times\ 10^{-3}$

h_{total} = (3.96 $\times\ 10^{-3} + 0.066\ )m$

= 0.06996 m

Pressure change : P1 – P2 = ΔP

$\Delta P = \rho g h_{total}$

= 1000 $\times 9.81 \times 0.066996$

= 686.3 N/m^2

Power required to maintain the flow :

P = $\Delta P Q$

= 686.3 $\times\ 1.33 \times 10^{-3}$

= 0.912779 Watts