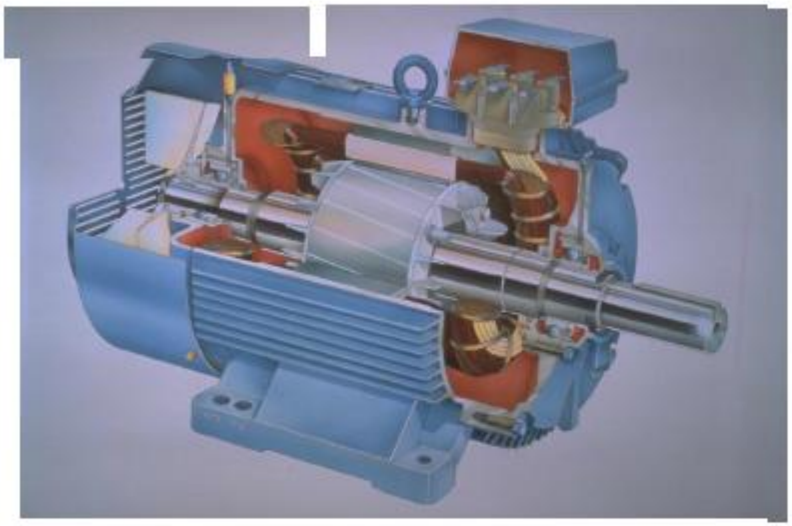
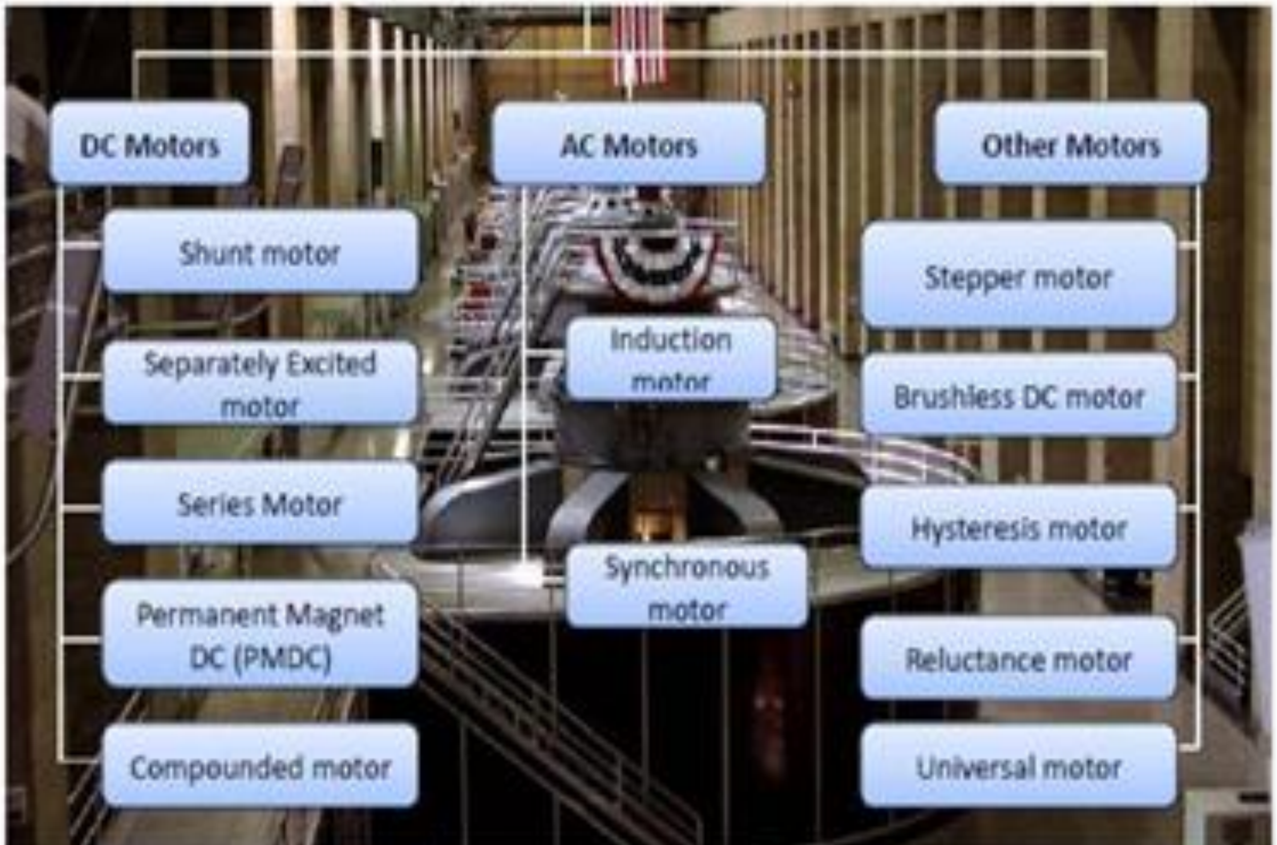
Induction Motor

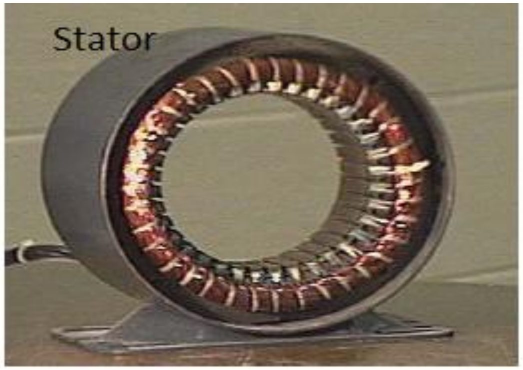


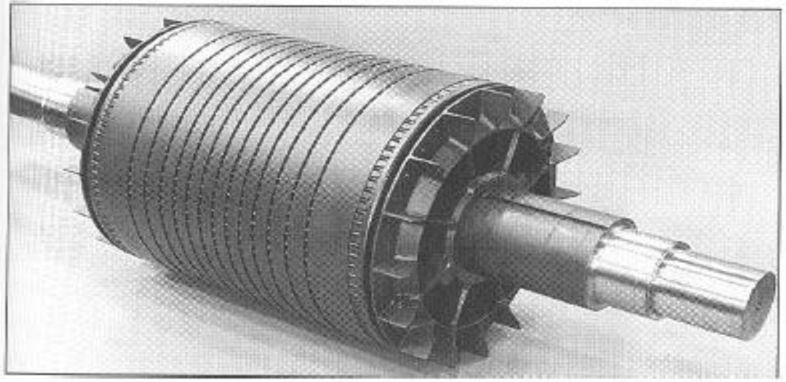
Types of Electric Motors



3- Phase induction machine construction

- 3 stator windings (uniformly distributed as in a synchronous generator)
- Two types of rotor:
  - Squirrel cage
  - Wound rotor (with slip rings)



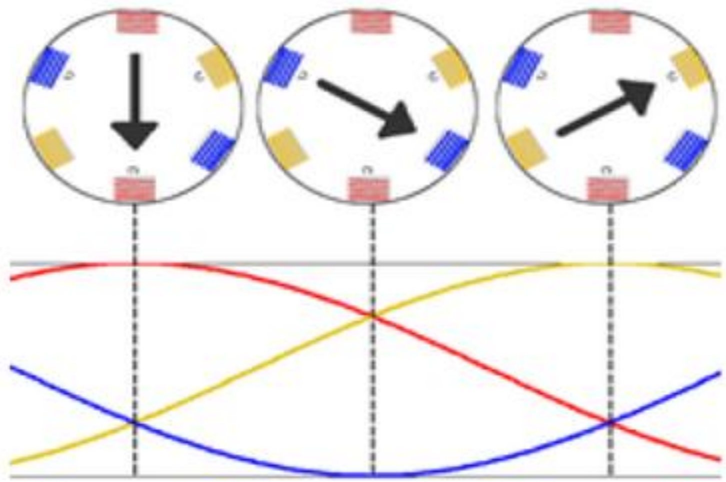


Squirrel cage rotor

Wound rotor (with slip rings)

The Rotating Magnetic Field

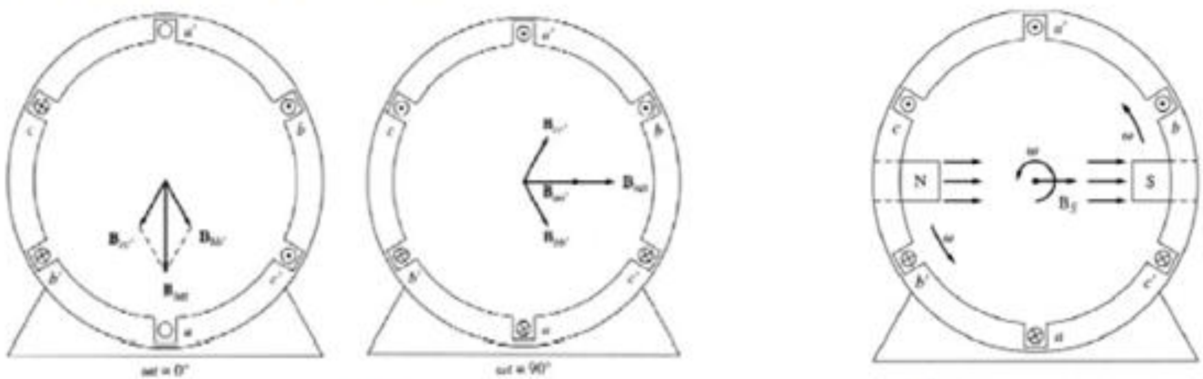
- The basic idea of an electric motor is to generate two magnetic fields: rotor magnetic field and stator magnetic field. The rotor will constantly be turning to align its magnetic field with the stator field.
- The 3-phase set of currents, each of equal magnitude and with a phase difference of 120°, flow in the stator windings and generate a rotating field will constant magnitude.



The Rotating Magnetic Field

The net magnetic field has a constant magnitude and rotates **counterclockwise** at the angular velocity ω.

The stator rotating magnetic field can be represented as a north pole and a south pole.



For a two pole machine, $$f_e\,(Hz) = f_m\,(rps) = \frac{1}{60} n_s\,(rpm)$$

For a p-pole machine, $$f_e\,(Hz) = \frac{p}{2} f_m\,(rps) = \frac{p}{120} n_s\,(rpm)$$

Principle of Operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction Motor Speed

At what speed will the induction motor run?

- Can the induction motor run at the synchronous speed, why?
- If rotor runs at the synchronous speed, then it will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed.
- When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced.

Induction Motor Speed

- So, the induction motor will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *slip speed*

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}}$$

Where s is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above by 100. Notice that the slip is a ratio and doesn't have units.

Induction Motors and Transformers

- Both induction motor and transformer works on the principle of induced voltage
  - Transformer: voltage applied to the primary windings produce an induced voltage in the secondary windings
  - Induction motor: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move
  - Due to the rotation of the rotor, the induced voltage in it does not have the same frequency of the stator voltage.

Rotor Frequency

- The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{p.n_{slip}}{120}$$

Where f_r = the rotor frequency (Hz)

p = number of stator poles

n_{slip} = slip speed (rpm)

$$f_r = \frac{p(n_{syn} - n_m)}{120} = \frac{p.s.n_{syn}}{120} = sf_e$$

Rotor Frequency

- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$\boxed{f_r = s\, f_e}$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency.
- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero.

Torque

- While the input to the induction motor is electrical power, its output is mechanical power and for that we should know some terms and quantities related to mechanical power.
- Any mechanical load applied to the motor shaft will introduce a torque on the motor shaft. This torque is related to the motor output power and the rotor speed

$$\boxed{\tau_{load} = \frac{P_{out}}{\omega_m} \quad N.m}$$ and $$\boxed{\omega_m = \frac{2\pi n_m}{60} \quad rad/s}$$

Horse Power

- Another unit used to measure mechanical power is the horse power.
- It is used to refer to the mechanical output power of the motor.
- Since we, as an electrical engineers, deal with watts as a unit to measure electrical power, there is a relation between horse power and watts:

$$\boxed{hp = 746\ watts}$$

Example 1

A 208 V, 10 hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?
4. What is the shaft torque of this motor at rated load?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

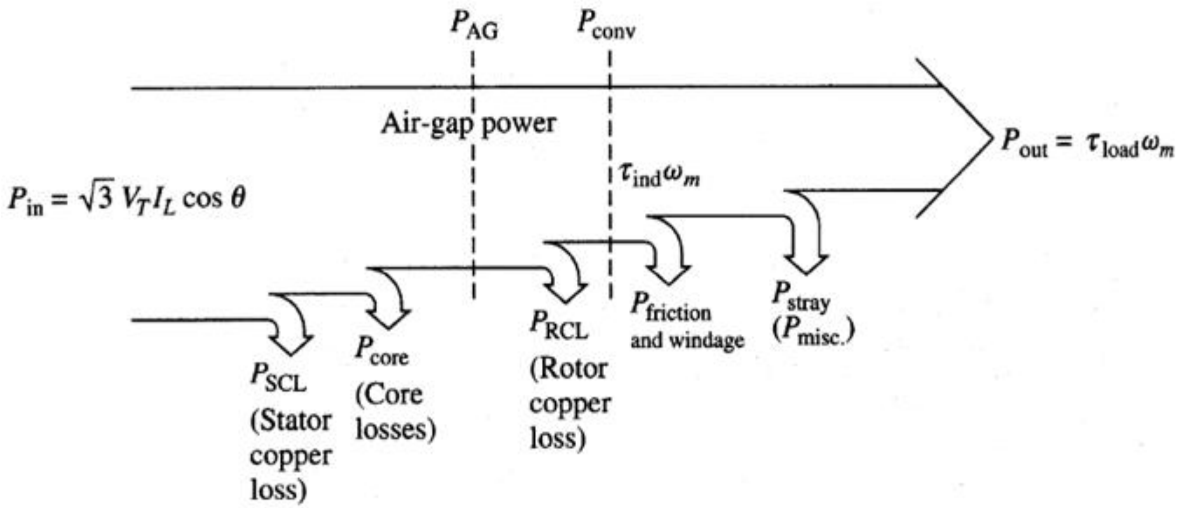
3. $$f_r = sf_e = 0.05\times 60 = 3 Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\, hp \times 746\, watt / hp}{1710\times 2\pi \times (1/60)} = 41.7\ N.m$$

Power losses in Induction Machines

- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2R_2$
- Core loss (P_{core})
- Mechanical power loss due to friction and windage
- How this power flow in the motor?

Power flow in Induction Motor



Power Relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3\, V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m}$$

Power Relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3\, V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3 I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3 I_2^2 \frac{R_2(1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

$$P_{conv} = (1-s) P_{AG}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s) P_{AG}}{(1-s)\omega_s}$$

Example 2

A 480-V, 60 Hz, 50-hp, three phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

1. The air-gap power P_{AG}.
2. The power converted P_{conv}.
3. The output power P_{out}.
4. The efficiency of the motor.

Solution

1. $$P_{in} = \sqrt{3} V_L I_L \cos\theta$$
$$= \sqrt{3} \times 480 \times 60 \times 0.85 = 42.4 \text{ kW}$$
$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4 - 2 - 1.8 = 38.6 \text{ kW}$$

2. $$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6 - \frac{700}{1000} = 37.9 \text{ kW}$$

3. $$P_{out} = P_{conv} - P_{F\&W}$$
$$= 37.9 - \frac{600}{1000} = 37.3 \text{ kW}$$

4. $$P_{out} = \frac{37.3}{0.746} = 50 \text{ hp}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{37.3}{42.4} \times 100 = 88\%$$

Example 3

A 460-V, 25-hp, 60 Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

R_1= 0.641Ω R_2= 0.332Ω

X_1= 1.106 Ω X_2= 0.464 Ω X_M= 26.3 Ω

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

1. Speed
2. Stator current
3. Power factor
4. P_{conv} and P_{out}
5. τ_{ind} and τ_{load}
6. Efficiency

Solution

1.
$$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$
$$n_m = (1-s) n_{sync} = (1-0.022) \times 1800 = 1760 \text{ rpm}$$

2.
$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464 = 15.1\angle 1.76^\circ \ \Omega$$
$$Z_f = \frac{1}{1/jX_M + 1/Z_2} = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$
$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ \ \Omega$$

Solution

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

$$I_1 = \frac{V_\phi}{Z_{tot}} = \frac{\frac{460\angle 0^\circ}{\sqrt{3}}}{14.07\angle 33.6^\circ} = 18.88\angle -33.6^\circ\ \text{A}$$

$$PF = \cos 33.6^\circ = 0.833 \quad \text{lagging}$$

$$P_{in} = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 460 \times 18.88 \times 0.833 = 12530\ \text{W}$$

3. $P_{SCL} = 3I_1^2 R_1 = 3(18.88)^2 \times 0.641 = 685\ \text{W}$

4. $P_{AG} = P_{in} - P_{SCL} = 12530 - 685 = 11845\ \text{W}$

Solution

$$P_{conv} = (1-s)P_{AG} = (1-0.022)(11845) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{F\&W} = 11585 - 1100 = 10485 \text{ W}$$

$$= \frac{10485}{746} = 14.1 \text{ hp}$$

5. $$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{11845}{2\pi \times 1800/60} = 62.8 \text{ N.m}$$

$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{10485}{2\pi \times 1760/60} = 56.9 \text{ N.m}$$

6. $$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{10485}{12530} \times 100 = 83.7\%$$

Example 4

A two-pole, 50-Hz induction motor supplies 15kW to a load at a speed of 2950 rpm.

1. What is the motor's slip?
2. What is the induced torque in the motor in N.m under these conditions?
3. What will be the operating speed of the motor if its torque is doubled?
4. How much power will be supplied by the motor when the torque is doubled?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{3000 - 2950}{3000} = 0.0167 \text{ or } 1.67\%$$

$\because$ no P_{f+W} given

2. $\therefore$ assume $P_{conv} = P_{load}$ and $\tau_{ind} = \tau_{load}$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{15 \times 10^3}{2950 \times \frac{2\pi}{60}} = 48.6 \text{ N.m}$$

Solution

3. In the low-slip region, the torque-speed curve is linear and the induced torque is direct proportional to slip. So, if the torque is doubled the new slip will be 3.33% and the motor speed will be

$$n_m = (1-s)n_{sync} = (1-0.0333)\times 3000 = 2900 \text{ rpm}$$

4. $$P_{conv} = \tau_{ind}\omega_m$$

$$= (2\times 48.6)\times(2900\times\frac{2\pi}{60}) = 29.5 \text{ kW}$$