2.3 STATIONARY MEDIA WITH SPECIFIED SURFACE CONCENTRATION

In the diffusion of gas from containers, there is diffusion of gas from inside to the outside without the metal molecules diffusing into the gas. In these cases the concentration of gas at the surfaces should be known. The solubility of the gas in the surface determines the concentration at the surface. These cases are similar to conduction through the medium. In these cases the temperature potential in conduction is replaced by concentration potential $(C_{a1} - C_{a2})$ for component A. The flow rate can be obtained as in the case of conduction. i.e

$$N_a = \frac{C_{a1} - C_{a2}}{R}$$, where R is the resistance of diffusion.

The resistance in the case of plane wall is;

$$R_p = \frac{L}{D_{ab} A}$$

(7)

For hollow cylindrical configuration is;

$$R_{cyl} = \frac{\ln(r_2 / r_1)}{2\pi D_{ab} L}$$

(8)

For hollow sphere is;

$$R_{sp} = \frac{1}{4\pi D_{ab}} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

(9)

These equations can be derived from the general equation in Cartesian, cylindrical and spherical coordinate systems.

Example 2: Hydrogen stored in a vessel diffuses through the steel wall of 20 mm thickness. The molar concentration at the inner surface is 2 kg mol/m³. At the other surface it is zero. Assuming plane wall condition and Dab = 0.26 × 10–12 m²/s, determine the mass of hydrogen diffused per 1 m²

Solution:

$$N_a = - D_{ab} \cdot \frac{C_{a2} - C_{a1}}{L} = D_{ab} \cdot \frac{C_{a1} - C_{a2}}{L} = D_{ab} \cdot \frac{C_{a1}}{L}$$

$$= 0.26 \times 10^{-12} \times \frac{2}{0.02} = 2.6 \times 10^{-11} \text{ kg mol/s m}^2.$$

For H_2, molecular weight is 2.

∴ mass diffused $= 2 \times 2.6 \times 10^{-11} = 5.2 \times 10^{-11} \text{ kg/m}^2\text{s}.$

3.0 DIFFUSION OF ONE COMPONENT INTO A STATIONARY COMPONENT OR UNIDIRECTIONAL DIFFUSION

In this case one of the components diffuses while the other is stationary. For steady conditions the mass diffused should be absorbed continuously at the boundary. In certain cases this is not possible. The popular example is water evaporating into air. In this case, as mentioned earlier, a bulk motion replaces the air tending to accumulate at the interface without being absorbed, causing an increase in the diffusion

rate. The diffusion equation for gases can be derived as (with 'a' as the diffusing medium and P = total pressure)

$$\frac{N_a}{A} = \frac{P}{\Re T} \cdot \frac{D}{(x_2 - x_1)} \cdot \ln\left(\frac{P - P_{a2}}{P - P_{a1}}\right)$$

For liquids (considering 'a' as diffusing medium)

$$\frac{N_a}{A} = \frac{D \cdot C}{(x_2 - x_1)} \cdot \ln\left(\frac{C - C_{a2}}{C - C_{a1}}\right)$$

3.1 UNSTEADY DIFFUSION

The equation for one dimensional unsteady diffusion is

$$\frac{\partial^2 C_a}{\partial x^2} = \frac{1}{D} \cdot \frac{\partial C_a}{\partial \tau}$$

(12)

This is similar to heat conduction equation with Ca and D replaced by T and α. All the solutions for various boundary conditions for unsteady conduction are applicable for mass transfer, except that the parameters are different.

4 CONVECTIVE MASS TRANSFER

When a medium deficient in a component flows over a medium having an abundance of the component, then the component will diffuse into the flowing medium. Diffusion in the opposite direction will occur if the mass concentration levels of the component are interchanged. In this case a boundary layer develops and at the interface mass transfer occurs by molecular diffusion (In heat flow at the interface, heat transfer is by conduction).

Velocity boundary layer is used to determine wall friction. Thermal boundary layer is used to determine convective heat transfer. Similarly concentration boundary layer is used to determine convective mass transfer. Figure 1 shows the flow of a mixture of components A and B with a specified constant concentration over a surface rich in component A. A concentration boundary layer develops. The concentration gradient varies from the surface to the free stream. At the surface the mass transfer is by diffusion. Convective mass transfer coefficient h_m is defined by the equation, where h_m has a unit of m/s.
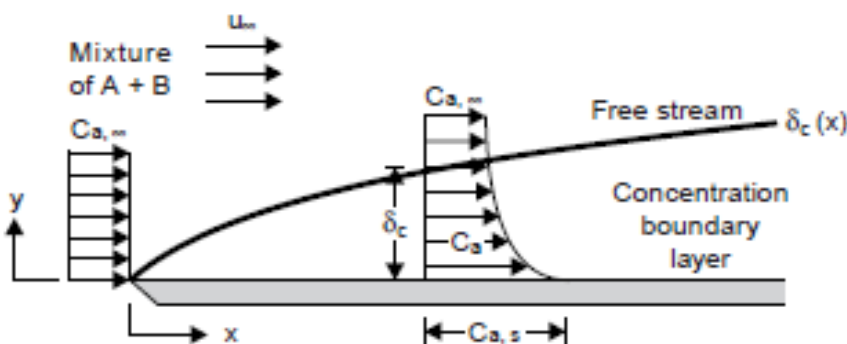

Figure 1: Species concentration boundary layer development on a flat plate.

$$\text{Moleflow} = h_m(C_{as} - C_{a\infty})$$

The condition for diffusion at the surface is given by

$$\text{Mole flow} = -D_{ab}\frac{\partial C_a}{\partial y}\bigg|_{y=0}$$

$$\therefore \qquad h_m = \frac{-D_{ab}\cdot\dfrac{\partial C_a}{\partial y}\bigg|_{y=0}}{C_{as} - C_{a\infty}}$$

In the above case, if mass flow is to be used then

$$h_m = \frac{-D_{ab}\cdot\dfrac{\partial \rho_a}{\partial y}\bigg|_{y=0}}{\rho_{as} - \rho_{a\infty}}$$

Similar to the momentum and energy equation, the mass concentration equation can be obtained as below:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\cdot\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$u\frac{\partial C_a}{\partial x} + v\frac{\partial C_a}{\partial y} = D_{ab}\frac{\partial^2 C_a}{\partial y^2}$$

By similarity the solutions for boundary layer thickness for connective mass transfer can be obtained. This is similar to the heat transfer by analogy. In this case, in the place of Prandtl number Schmidt number defined by

$$Sc = v/D_{ab}$$

Nondimensionalising the equation leads to the condition as below:

$$\delta_m = f(Re,\ Sc)$$
$$Sh = f(Re,\ Sc)$$

where Sherwood number Sh is defined as

$$Sh = \frac{h_m x}{D_{ab}}.$$

In the laminar region flow over plate;

$$\delta_{mx} = \frac{5x}{Re_x^{1/2}}\cdot Sc^{-1/3}$$

$$Sh_x = \frac{h_{mx} x}{D_{ab}} = 0.332\ Re_x^{1/2}\ Sc^{1/3}$$

$$\overline{Sh_L} = \frac{h_m L}{D_{ab}} = 0.664\ Re_2^{1/2}\ Sc^{1/3}$$

In the turbulent region $Re > 5 \times 10_5$,

$$\delta_m = \delta_v$$
$$Sh_x = 0.0296 \, Re_x^{\,0.8} \, Sc^{1/3}$$
$$\overline{Sh}_L = 0.037 \, Re_L^{\,0.8} \, Sc^{1/3}$$

For flow through tubes,

In the laminar region, $Re < 2000$

For uniform wall mass concentration,

$$Sh = 3.66$$

For uniform wall mass flux

$$Sh = 4.36$$

For turbulent region,

$$Sh = 0.023 \, Re^{0.83} \, Sc^{1/3}$$

Example 3: A pipe carrying ammonia at 1 bar and 40°C is vented to a large tank containing dry air at 1 bar and 40°C to avoid pressure build up. The pipe is 5 mm in dia and 5 m long. Determine the rate of diffusion of air into the ammonia stream. $D = 0.28 \times 10\text{–}4$ m²/s. Solution: The partial pressure of air at the end of pipe is 1 bar and at the connection with ammonia tank it is zero.

Solution:

$$N_a = \frac{\pi \times 0.005^2}{4} \times \frac{(10^5 - 0)}{5} \times \frac{0.28 \times 10^{-4}}{8315 \times 313}$$

$$= 4.225 \times 0^{-12} \text{ kg mol/s}$$

or in terms of mass $\quad N_a \times M_a$ gives 1.263×10^{-10} kg/s.

Example 4: A well is 40 m deep and 9 m dia and the atmospheric temperature is 25°C. The air at the top is having a relative humidity of 50%. Determine the rate of diffusion of water vapour through the well $D = 2.58 \times 10^{-5}$ m²/s.

Solution: This is a situation of diffusion into a stationary medium. The partial pressure is equal to saturation pressure at 25°C = 0.03169 bar. At the top RH = 50%.
∴ Partial pressure = 0.5 × 0.03169 bar.
If mass is to be calculated use the gas constant for the gas instead of universal gas constant.

$$m_a = A \cdot \frac{DP}{R_a \, T(x_2 - x_1)} \cdot \ln \frac{P - P_{b2}}{P - P_{b1}}$$

$$R_a = \frac{8315}{18} \text{ J/kgK}, \; T = 298 \text{ K}., \; x_2 - x_1 = 40 \text{ m}$$

$$A = \frac{\pi \times 9^2}{4}, \, P = 10^5 \text{ N/m}^2$$

$$m_a = \frac{\pi \times 9^2}{4} \times \frac{2.58 \times 10^{-5} \times 18}{8315 \times 298 \times 40} \times 10^5 \cdot \ln \frac{10^5(1 - 0.5 \times 0.03169)}{10^5(1 - 0.03169)}$$

$$= 4.84 \times 10^{-7} \text{ kg/s (low because of large depth)}.$$

Reference:

Fundamentals of Heat and Mass Transfer by C. P. Kothandaraman