Collection of Fluid Power equations – Basic course

Quantity	Equation
Pressure [Pa], external load	$p = \frac{F}{A}$
Input quantities	F = force [N] A = area [m^2]
Pressure [Pa], hydrostatic	$p_h = \rho \cdot g \cdot h$
Input quantities	ρ = fluid density [kg/m^3] g = 9,81 m/s^2 h = distance from free fluid surface [m]
Pressure [Pa], total static	$p_{st} = \frac{F}{A} + p_h + p_{am}$
Input quantities	p_{am} = ambient pressure [Pa]
Pressure [Pa], absolute	$p_{abs} = p_{am} + p_{aux}$
Input quantities	p_{am} = ambient pressure [Pa] p_{aux} = pressure induced by external and internal loading [Pa]
Bernoulli equation, dynamic total pressure [Pa]	$p_{dyn} = p + \rho \cdot g \cdot z + \frac{\rho \cdot v^2}{2} = \text{constant}$
Input quantities	p = static pressure [Pa] ρ = fluid density [kg/m^3] g = 9,81 m/s^2 z = elevation [m] v = flow velocity [m/s]
Energy equation [Pa]	$p_1 + \rho \cdot g \cdot z_1 + \frac{\rho \cdot v_1^2}{2} = p_2 + \rho \cdot g \cdot z_2 + \frac{\rho \cdot v_2^2}{2} + p_s$
Input quantities	p = static pressure [Pa] ρ = fluid density [kg/m^3] g = 9,81 m/s^2 z = elevation [m] v = flow velocity [m/s] p_s = pressure loss [Pa]

Quantity	Equation
Hydraulic diameter [m]	$D_{\mathrm{H}} = \frac{4 \cdot A}{L_{\mathrm{A}}}$
Input quantities	A = cross-sectional area of flow channel [m^2] L_{A} = wetted perimeter of area A [m]
Reynold’s number []	$Re = \frac{v \cdot D_{\mathrm{H}}}{\nu}$
Input quantities	v = flow velocity [m/s] ν = kinematic viscosity [m^2/s] D_{H} = hydraulic diameter [m]
Pipe friction coefficient (Darcy-Weisbach friction coefficient) in laminar flow case []	$\lambda = \frac{64}{Re}$
Input quantities	Re = Reynold’s number []
Pressure loss in straight pipe sections [Pa]	$\Delta p = \lambda \cdot \frac{l}{D_{\mathrm{H}}} \cdot \frac{\rho}{2} \cdot v^2$
Input quantities	λ = Darcy-Weisbach friction coefficient [] l = pipe length [m] D_{H} = hydraulic diameter [m] ρ = fluid density [kg/m^3] v = average flow velocity in flow channel’s cross-section [m/s]
Pressure loss in case of change in flow direction or velocity [Pa]	$\Delta p = \zeta \cdot \frac{\rho}{2} \cdot v^2$
Input quantities	ζ = resistance coefficient [] ρ = fluid density [kg/m^3] v = average flow velocity in flow channel’s cross-section [m/s]
Total pressure loss in piping system [Pa]	$\Delta p_{\mathrm{t}} = \sum_{i=1}^{N1} \lambda_i \cdot \frac{l}{D_{\mathrm{H},i}} \cdot \frac{\rho_i}{2} \cdot v_i^2 + \sum_{j=1}^{N2} \zeta_j \cdot \frac{\rho_j}{2} \cdot v_j^2$
Input quantities	λ = Darcy-Weisbach friction coefficient [] l = pipe length [m] D_{H} = hydraulic diameter [m] ρ = fluid density [kg/m^3] v = average flow velocity in flow channel’s cross-section [m/s] ζ = resistance coefficient [] i, j = index []

Quantity	**Equation**
Effect of viscosity on pressure losses of components [Pa]	$\Delta p_2 \approx \left(\frac{\nu_2}{\nu_1}\right)^{0,25} \cdot \Delta p_1$
Input quantities	ν_1 = kinematic viscosity 1 [m^2/s] ν_2 = kinematic viscosity 2 [m^2/s] Δp_1 = pressure loss corresponding viscosity 1 [Pa]
Total pressure level [Pa]	$p_t = p_{ex} + \Delta p_t$
Input quantities	p_{ex} = pressure induced by external loading [Pa] Δp_t = total pressure losses [Pa]
Velocity of pressure wave in medium [m/s]	$c = \sqrt{\frac{K_e}{\rho}}$
Input quantities	K_e = system'effective bulk modulus [N/m^2] ρ = fluid density [kg/m^3]
Critical closing time of valve [s]	$t_{cr} = \frac{2 \cdot l}{c}$
Input quantities	l = distance between birth and reflection points of pressure wave [m] c = velocity of pressure wave in medium [m/s]
Pressure shock induced pressure rise in piping system, case: rapid valve closing ($t_c < t_{cr}$) [Pa]	$\Delta p_{max} = \rho_0 \cdot c \cdot v$
Input quantities	ρ_0 = fluid density before pressure shock [kg/m^3] c = velocity of pressure wave in medium [m/s] v = average flow velocity before valve closure [m/s] t_c = valve closing time [s]
Pressure shock induced pressure rise in piping system, case: slow valve closing ($t_c > t_{cr}$) [Pa]	$\Delta p_{max} = \frac{2 \cdot l \cdot \rho_0 \cdot v}{t_c}$
Input quantities	l = distance between birth and reflection points of pressure wave [m] ρ_0 = fluid density before pressure shock [kg/m^3] v = average flow velocity before valve closure [m/s] t_c = valve closing time [s]

Quantity	**Equation**
Pressure shock induced pressure rise in cylinder volume [Pa]	$\Delta p_{\max} = \dfrac{K_e \cdot A}{V_0} \cdot v \cdot \sqrt{\dfrac{m}{k_H}}$
Input quantities	K_e = effective bulk modulus of closed volume [N/m^2] A = piston area on side of closed volume [m^2] V_0 = closed volume at the moment of valve closure [m^3] v = cylinder velocity before stopping [m/s] m = stopped total mass [kg] k_H = hydraulic spring constant [N/m] *By the "closed volume" here is meant the summed volume of cylinder chamber and piping between the cylinder and the valve on the side of direction of movement.*
Pressure shock induced pressure rise in hydraulic motor volume [Pa]	$\Delta p_{\max} = \omega \cdot \sqrt{\dfrac{K_e \cdot J}{V_0}}$
Input quantities	K_e = effective bulk modulus of closed volume [N/m^2] ω = motor's angular velocity before stopping [rad/s] J = moment of inertia of rotating parts of motor and load reduced on motor axle [kgm^2] V_0 = closed volume at the moment of valve closure [m^3] *By the "closed volume" here is meant the summed volume of 0,5·motor displacement and piping between the motor and the valve on the side of direction of rotation.*
Total pressure level in pressure shock [Pa]	$p_t = p_{sys,st} + \Delta p_{\max}$
Input quantities	$p_{sys,st}$ = system's static pressure level [Pa] $\Delta p_{\max}$ = pressure rise induced by pressure shock [Pa]

Quantity	Equation
Converting / reducing pressure over cylinder's piston [Pa]	$p_{\text{converted}} = \frac{A_{\text{tobeconverted}}}{A_{\text{converted}}} \times p_{\text{tobeconverted}}$
Input quantities	$A_{\text{tobeconverted}}$ = piston area on the side of the pressure to be converted [m^2] $A_{\text{converted}}$ = piston area on the side of the converted pressure [m^2] $p_{\text{tobeconverted}}$ = pressure to be converted [Pa]
Flow rate [m^3/s]	$q_V = A \cdot v$
Input quantities	A = cross-sectional area of flow channel, perpendicular to flow [m^2] v = average flow velocity in flow channel's cross-section [m/s]
Mass flow rate [kg/s]	$q_m = \rho \cdot \dot{V} + V \cdot \dot{\rho}$
Input quantities	ρ = fluid density [kg/m^3] $\dot{V}$ = flow rate [m^3/s] V = volume [m^3] $\dot{\rho}$ = change in fluid density [kg/m^3s]
Kirchhoff's I law [m^3/s]	$\sum_{i=1}^{N1} q_{V,i} = \sum_{j=1}^{N2} q_{V,j}$
Input quantities	$q_{V,i}$ = incoming flow rate [m^3/s] $q_{V,j}$ = outgoing flow rate [m^3/s] i, j = index []
Throttle equation [m^3/s]	$q_V = C_q \cdot A \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$
Input quantities	C_q = flow coefficient [] A = cross-sectional flow area of throttle [m^2] Δp = pressure difference over the throttle [Pa] ρ = fluid density [kg/m^3]

Quantity	Equation
Flow rate in laminar pipe flow [m³/s]	$q_{\mathrm{V}} = \dfrac{\pi \cdot d^4}{128 \cdot \eta \cdot l} \cdot (p_1 - p_2)$
Input quantities	d = pipe's inner diameter [m] η = dynamic viscosity [Pa·s] l = pipe length [m] p_1 = pressure at upstream point 1 [Pa] p_2 = pressure at downstream point 2 [Pa]
Flow rate in laminar rectangular gap flow [m³/s]	$q_{\mathrm{V}} = \dfrac{b \cdot h^3}{12 \cdot \eta \cdot l} \cdot (p_1 - p_2)$
Input quantities	b = gap width [m] h = gap height [m] η = dynamic viscosity [Pa·s] d = gap length [m] p_1 = pressure at upstream point 1 [Pa] p_2 = pressure at downstream point 2 [Pa]
Flow rate in laminar annular gap flow [m³/s]	$q_{\mathrm{V}} = \dfrac{\pi \cdot d \cdot h^3}{12 \cdot \eta \cdot l} \cdot \left[1 + 1{,}5 \cdot \left(\dfrac{e}{h}\right)^2\right] \cdot (p_1 - p_2)$
Input quantities	d = outer inner diameter of flow channel [m] h = gap height [m] η = dynamic viscosity [Pa·s] d = gap length [m] e = eccentricity [] p_1 = pressure at upstream point 1 [Pa] p_2 = pressure at downstream point 2 [Pa]
Kinematic viscosity [m²/s]	$\nu = \dfrac{\eta}{\rho}$
Input quantities	η = dynamic viscosity [Pa·s] ρ = fluid density [kg/m³]
Density as function of temperature [kg/m³]	$\rho_{\theta} = \dfrac{\rho_{15}}{1 + \alpha \cdot (\theta - 15)}$
Input quantities	ρ_{15} = fluid density at 15 °C [kg/m³] θ = temperature [°C] α = thermal expansion coefficient of volume [1/°C]

Quantity	Equation
Density as function of pressure [kg/m³]	$\rho_{p2} = \dfrac{\rho_{p1}}{1 - \chi_p \cdot (p_2 - p_1)}$
Input quantities	ρ_{p1} = fluid density at pressure p_1 [kg/m³] p_1 = initial pressure [Pa] p_2 = final pressure [Pa] χ_p = compressibility coefficient [m²/N]
Thermal capacity [J/K]	$C_\theta = \sum_{i=1}^{N} m_i \cdot c_{p,i}$
Input quantities	m = mass [kg] c_p = specific heat [J/kgK] i = index []
Heat transfer ability aka cooling ability of system [W/K]	$B_\theta = \sum_{i=1}^{N} C_{U,i} \cdot A_i$
Input quantities	C_U = thermal transmittance [W/m²K] A = heat transmitting area [m²] i = index []
Final temperature of system [K]	$\theta_e = \theta_0 + \dfrac{P_s}{B_\theta}$
Input quantities	θ_0 = system’s initial temperature at time $t = 0$ [K] P_s = system’s average power loss [W] B_θ = system’s heat transfer ability [W/K]
Bulk modulus of hollow cylindrical part [N/m²]	$K = \dfrac{E_m \cdot s}{d}$
Input quantities	E_m = part’s modulus of elasticity [N/m²] s = part’s wall thickness [m] d = part’s inner diameter [m]
Bulk modulus of free air in adiabatic change of state [N/m²]	$K_a = 1{,}4 \cdot p$
Input quantities	p = system’s pressure level [Pa]

Quantity	Equation
Effective bulk modulus [N/m^2]	$\frac{1}{K_\mathrm{e}} = \frac{1}{K_\mathrm{f}} + \sum_{i=1}^{N1}\left(\frac{V_{\mathrm{c},i}}{V_\mathrm{t}} \cdot \frac{1}{K_{\mathrm{c},i}}\right) + \sum_{j=1}^{N2}\left(\frac{V_{\mathrm{p},j}}{V_\mathrm{t}} \cdot \frac{1}{K_{\mathrm{p},j}}\right) + \sum_{k=1}^{N3}\left(\frac{V_{\mathrm{h},k}}{V_\mathrm{t}} \cdot \frac{1}{K_{\mathrm{h},k}}\right) + \frac{V_\mathrm{a}}{V_\mathrm{t}} \cdot \frac{1}{K_\mathrm{a}}$
Input quantities	K_f = bulk modulus of fluid [N/m^2] V_t = total volume of pressurized system [m^3] V_c = volume of single cylinder [m^3] K_c = bulk modulus of single cylinder [N/m^2] V_p = volume of single pipe [m^3] K_p = bulk modulus of single pipe [N/m^2] V_h = volume of single hose [m^3] K_h = bulk modulus of single hose [N/m^2] V_a = volume of free air [m^3] K_a = bulk modulus of air [N/m^2] i, j, k = index []
Volume change induced by compressibility [m^3]	$\Delta V = \frac{1}{K_\mathrm{e}} \cdot V_0 \cdot \Delta p$
Input quantities	K_e = system's effective bulk modulus [N/m^2] V_0 = system's initial volume [m^3] Δp = pressure change in fluid [Pa]
Impulse [kgm/s]	$I_\mathrm{F} = m \cdot \Delta v$
Input quantities	m = moving mass [kg] Δv = change in velocity [m/s]
Flow force [N]	$\bar{F}_\mathrm{q} = \rho \cdot q_\mathrm{V} \cdot \bar{v}$
Input quantities	ρ = fluid density [kg/m^3] q_V = flow rate [m^3/s] $\bar{v}$ = flow velocity vector [m/s]

Quantity	**Equation**
Hydraulic power [W]	$P = q_V \cdot p$
Input quantities	q_V = flow rate [m^3/s] p = pressure [Pa]
Hydraulic power loss [W]	$P_s = q_V \cdot p_s$
Input quantities	q_V = flow rate [m^3/s] p_s = pressure loss [Pa]
Input power [W]	$P_{in} = P_{out} + P_s$
Input quantities	P_{out} = output power [W] P_s = power loss [W]
Total efficiency []	$\eta_t = \frac{P_{out}}{P_{in}}$
Input quantities	P_{out} = output power [W] P_{in} = input power [W]
Total efficiency []	$\eta_t = \eta_v \cdot \eta_{hm}$
Input quantities	η_v = volumetric efficiency [] η_{hm} = hydromechanical efficiency []
Total efficiency of work cycle []	$\eta_{t,wc} = \frac{P_{out,wc}}{P_{in,wc}} = \frac{\sum_{i=1}^{N} P_{in,i} \cdot \eta_{t,i} \cdot t_i}{\sum_{i=1}^{N} P_{in,i} \cdot t_i}$
Input quantities	$P_{out,wc}$ = utility power during work cycle [W] $P_{in,wc}$ = power usage during work cycle [W] $P_{in,i}$ = input power of a single phase of work cycle [W] $\eta_{t,i}$ = total efficiency of a single phase of work cycle [] t_i = duration of a single phase of work cycle [s] i = index []

Quantity	**Equation**
Pump flow [m³/s]	$q_V = n \cdot V_g \cdot \eta_v$
Input quantities	n = rotational velocity [r/s] V_g = geometric displacement, swept volume [m^3/r] η_v = volumetric efficiency []
Pump drive torque [Nm]	$T = \frac{\Delta p \cdot V_g}{2 \cdot \pi \cdot \eta_{hm}}$
Input quantities	Δp = pressure difference between pump’s inlet and outlet [Pa] V_g = geometric displacement, swept volume [m^3/r] η_{hm} = hydromechanical efficiency []
Pump drive power [W]	$P = T \cdot \omega = \frac{q_V \cdot \Delta p}{\eta_t}$
Input quantities	T = pump drive torque [Nm] ω = angular velocity of pump axle [rad/s] q_V = pump flow [m^3/s] Δp = pressure difference between pump’s inlet and outlet [Pa] η_t = total efficiency []
Rotational velocity of hydraulic motor [r/s]	$n = \frac{q_V \cdot \eta_v}{V_g}$
Input quantities	q_V = flow rate [m^3/s] V_g = geometric displacement, swept volume [m^3/r] η_v = volumetric efficiency []
Motor torque [Nm]	$T = \frac{V_g \cdot \Delta p \cdot \eta_{hm}}{2 \cdot \pi}$
Input quantities	V_g = geometric displacement, swept volume [m^3/r] Δp = pressure difference between motor’s inlet and outlet [Pa] η_{hm} = hydromechanical efficiency []
Motor power [W]	$P = q_V \cdot \Delta p \cdot \eta_t = T \cdot \omega$
Input quantities	q_V = flow rate [m^3/s] Δp = pressure difference between motor’s inlet and outlet [Pa] η_t = total efficiency [] T = motor’s torque [Nm] ω = angular velocity of motor axle [rad/s]

Quantity	Equation
Angular velocity [rad/s]	$\omega = 2 \cdot \pi \cdot n$
Input quantities	n = rotational velocity [r/s]
Cylinder velocity [m/s]	$v = \frac{q_{\mathrm{V,in}} \cdot \eta_{\mathrm{v}}}{A_{\mathrm{in}}}$
Input quantities	$q_{\mathrm{V,in}}$ = input flow rate to cylinder [m^3/s] A_{in} = piston area on the input flow side [m^2] η_{v} = volumetric efficiency []
Cylinder force [N]	$F = \left(p_{\mathrm{in}} \cdot A_{\mathrm{in}} - p_{\mathrm{out}} \cdot A_{\mathrm{out}}\right) \cdot \eta_{\mathrm{hm}}$
Input quantities	p_{in} = pressure on the input flow side of cylinder [Pa] p_{out} = pressure on the output flow side of cylinder [Pa] A_{in} = piston area on the input flow side [m^2] A_{out} = piston area on the output flow side [m^2] η_{hm} = hydromechanical efficiency []
Cylinder power, mechanical [W]	$P = q_{\mathrm{V,in}} \cdot \left(p_{\mathrm{in}} - \frac{A_{\mathrm{out}}}{A_{\mathrm{in}}} \cdot p_{\mathrm{out}}\right) \cdot \eta_{\mathrm{t}} = F \cdot v$
Input quantities	$q_{\mathrm{V,in}}$ = input flow rate to cylinder [m^3/s] p_{in} = pressure on the input flow side of cylinder [Pa] p_{out} = pressure on the output flow side of cylinder [Pa] A_{in} = piston area on the input flow side [m^2] A_{out} = piston area on the output flow side [m^2] η_{t} = total efficiency [] F = cylinder force [N] v = cylinder velocity [m/s]
Cylinder's allowable loading	$F = \frac{\pi^2 \cdot E_{\mathrm{m}} \cdot I}{C_{\mathrm{n}} \cdot l_{\mathrm{R}}^2}$
Input quantities	E_{m} = modulus of elasticity [N/m^2] I = area moment of inertia [m^4] C_{n} = safety coefficient [] l_{R} = reduced length [m]

Quantity	**Equation**
Nominal volume of pressure accumulator [m^3]	$V_1 = \dfrac{\Delta V}{\left[\left(\dfrac{p_1}{p_2}\right)^{\frac{1}{k}} - \left(\dfrac{p_1}{p_3}\right)^{\frac{1}{k}}\right]}$
Input quantities	ΔV = fluctuating fluid volume [m^3] p_1 = gas precharge pressure [Pa] p_2 = minimum working pressure [Pa] p_3 = maximum working pressure [Pa] k = polytropic constant []
Filtration ratio []	$\beta_x = \dfrac{N_1}{N_2}$
Input quantities	N_1 = number of particles (size x) upstream of filter [] N_2 = number of particles (size x) downstream of filter []
Filter efficiency []	$S_x = \left(1 - \dfrac{1}{\beta_x}\right) \times 100\%$
Input quantities	β_x = filtration ratio for particle size x []