3.3. NEWTONIAN FLUIDS

3.3.1. Shearing characteristics of a Newtonian fluid

As a fluid is deformed because of flow and applied external forces, frictional effects are exhibited by the motion of molecules relative to each other. The effects are encountered in all fluids and are due to their *viscosities*. Considering a thin layer of fluid between two parallel planes, distance y apart as shown in Figure 3.4 with the lower plane fixed and a shearing force F applied to the other, since fluids deform continuously under shear, the upper plane moves at a steady velocity u_x relative to the fixed lower plane. When conditions are steady, the force F is balanced by an internal force in the fluid due to its viscosity and the shear force per unit area is proportional to the velocity gradient in the fluid, or:

$$\frac{F}{A} = R_y \propto \frac{u_x}{y} \propto \frac{du_x}{dy} \tag{3.2}$$

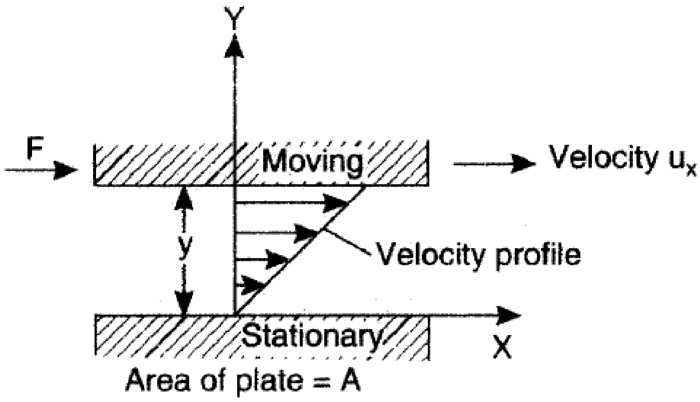

Figure 3.4. Shear stress and velocity gradient in a fluid

R is the shear stress in the fluid and du_x/dy is the velocity gradient or the rate of shear. It may be noted that R corresponds to τ used by many authors to denote shear stress; similarly, shear rate may be denoted by either du_x/dy or $\dot{\gamma}$. The proportionality sign may be replaced by the introduction of the proportionality factor μ, which is the coefficient of viscosity, to give:

$$R_y = \pm\mu\frac{du_x}{dy} \tag{3.3}$$

A *Newtonian* fluid is one in which, provided that the temperature and pressure remain constant, the shear rate increases linearly with shear stress over a wide range of shear rates. As the shear stress tends to retard the fluid near the centre of the pipe and accelerate the slow moving fluid towards the walls, at any radius within the pipe it is acting simultaneously in a negative direction on the fast moving fluid and in the positive direction on the slow moving fluid. In strict terms equation 3.3 should be written with the incorporation of modulus signs to give:

$$\mu = \frac{|R_y|}{|\mathrm{d}u_x/\mathrm{d}y|} \tag{3.4}$$

The viscosity strongly influences the shear stresses and hence the pressure drop for the flow. Viscosities for liquids are generally two orders of magnitude greater than for gases at atmospheric pressure. For example, at 294 K, $\mu_{\text{water}} = 1.0 \times 10^{-3}$ N s/m^2 and $\mu_{\text{air}} = 1.8 \times 10^{-5}$ N s/m^2. Thus for a given shear rate, the shear stresses are considerably greater for liquids. It may be noted that with increase in temperature, the viscosity of a liquid decreases and that of a gas increases. At high pressures, especially near the critical point, the viscosity of a gas increases with increase in pressure.

Calculation of pressure drop for liquid flowing in a pipe

For the flow of a fluid in a pipe of length l and diameter d, the total frictional force at the walls is the product of the shear stress R and the surface area of the pipe ($R\pi dl$). This frictional force results in a change in pressure ΔP_f so that for a horizontal pipe:

$$R\,\pi\,d\,l = -\Delta P_f \pi \frac{d^2}{4} \tag{3.15}$$

or:

$$-\Delta P_f = 4R\frac{l}{d} = 4\frac{R}{\rho u^2}\frac{l}{d}\rho u^2 = 4\phi\frac{l}{d}\rho u^2 \tag{3.16}$$

and:

$$\phi = \frac{R}{\rho u^2} = \frac{-\Delta P d}{4l\rho u^2} \tag{3.17}$$

The head lost due to friction is then:

$$h_f = \frac{-\Delta P_f}{\rho g} = 4\frac{R}{\rho u^2}\frac{l}{d}\frac{u^2}{g} \tag{3.18}$$

The energy dissipated per unit mass F is then given by equation 3.19:

$$F = \frac{-\Delta P_f}{\rho} = 4\frac{R}{\rho u^2}\frac{l}{d}u^2 = 4\phi\frac{l}{d}u^2 \tag{3.19}$$

To calculate $-\Delta P_f$ it is therefore necessary to evaluate e/d and obtain the corresponding value of $\phi = R/\rho u^2$ from a knowledge of the value of *Re*. This value of ϕ is then used in equation 3.16 to give $-\Delta P_f$ or the head loss due to friction h_f as:

$$h_f = \frac{-\Delta P_f}{\rho g} = 4\phi\frac{l}{d}\frac{u^2}{g} = 8\phi\frac{l}{d}\frac{u^2}{2g} \tag{3.20}$$

With the friction factors used by Moody and Fanning, f' and f respectively, the head loss due to friction is obtained from the following equations:

Moody:
$$h_f = f'\frac{l}{d}\frac{u^2}{2g} \tag{3.21}$$

Fanning:
$$h_f = 4f\frac{l}{d}\frac{u^2}{2g} \tag{3.22}$$

The energy dissipated per unit mass due to the irreversibility of the process is given by $F = -\Delta P_f/\rho = 4\phi(l/d)u^2$ (equation 3.19).

If it is necessary to calculate the flow in a pipe where the pressure drop is specified, the velocity u is required but the Reynolds number is unknown, and this approach cannot be used to give $R/\rho u^2$ directly. One alternative here is to estimate the value of $R/\rho u^2$ and calculate the velocity and hence the corresponding value of *Re*. The value of $R/\rho u^2$ is then determined and, if different from the assumed value, a further trial becomes necessary.

An alternative approach to this problem is to use a friction group formed by combining ϕ and *Re* as follows:

$$\phi Re^2 = \frac{R}{\rho u^2}\left(\frac{\rho u d}{\mu}\right)^2 = \frac{R d^2 \rho}{\mu^2} = \frac{-\Delta P_f d^3 \rho}{4 l \mu^2} \tag{3.23}$$

If *Re* is plotted as a function of ϕRe^2 and e/d as shown in Figure 3.8, the group $(-\Delta P_f d^3 \rho/4l\mu^2) = \phi Re^2$ may be evaluated directly as it is independent of velocity. Hence *Re* may be found from the graph and the required velocity $\left(u = \dfrac{Re\mu}{\rho d}\right)$ obtained. Similarly, if the diameter of pipe is required to transport fluid at a mass rate of flow G with a given fall in pressure, the following group, which is independent of d, may be used:

$$\left(\frac{R}{\rho u^2}\right)\left(\frac{u d \rho}{\mu}\right)^{-1} = \phi Re^{-1} = \frac{-\Delta P_f \mu}{4\rho^2 u^3 l} \tag{3.24}$$

Effect of roughness of pipe surfaces

The estimation of the roughness of the surface of the pipe often presents considerable difficulty. The use of an incorrect value is not usually serious, however, even for turbulent

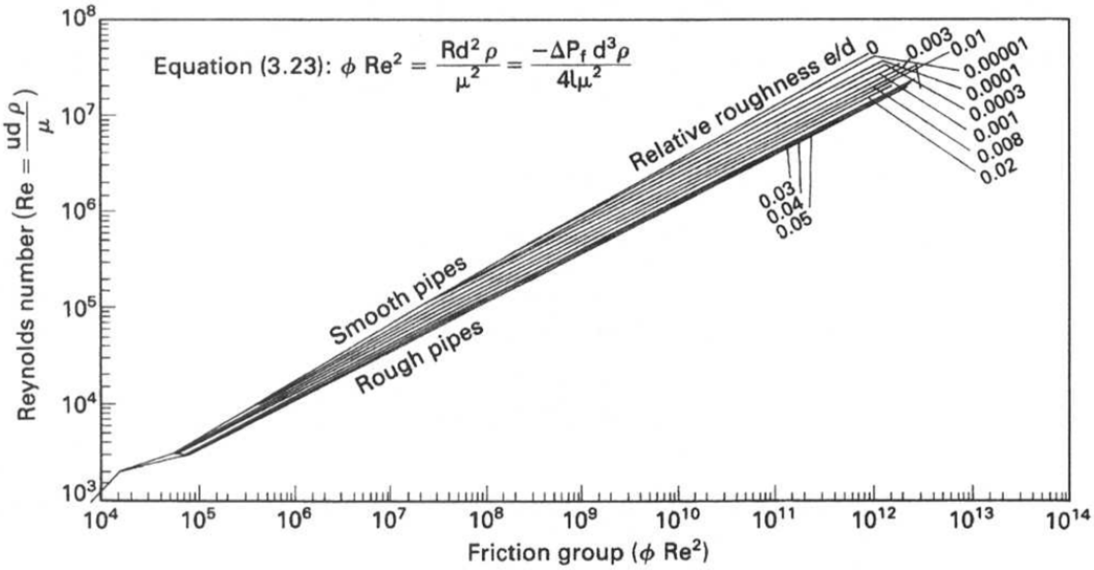

Figure 3.8. Pipe friction chart ϕRe^2 versus Re for various values of e/d (also see fold-out in the Appendix)

flow at low Reynolds numbers because the pressure drop is not critically dependent on the roughness in this region. However, at high values of Reynolds number, the effect of pipe roughness is considerable, as may be seen from the plot of $R/\rho u^2$ against the Reynolds number shown in Figure 3.7. The values of the absolute roughness have been measured for a number of materials and typical data are given in Table 3.1. Where the value for the pipe surface in question is not given, it is necessary to estimate an approximate value based on the available data. Where pipes have become corroded, the value of the roughness is commonly increased, up to tenfold.

Values of roughness applicable to materials used in the construction of open channels are also included in Table 3.1.

Table 3.1. Values of absolute roughness e

	(ft)	(mm)
Drawn tubing	0.000005	0.0015
Commercial steel and wrought-iron	0.00015	0.046
Asphalted cast-iron	0.0004	0.12
Galvanised iron	0.0005	0.15
Cast-iron	0.00085	0.26
Wood stave	0.0006–0.003	0.18–0.9
Concrete	0.001–0.01	0.3–3.0
Riveted steel	0.003–0.03	0.9–9.0

Example 3.1

Ninety-eight per cent sulphuric acid is pumped at 4.5 tonne/h (1.25 kg/s) through a 25 mm diameter pipe, 30 m long, to a reservoir 12 m higher than the feed point. Calculate the pressure drop in the pipeline.

$$\text{Viscosity of acid} = 25 \text{ mN s/m}^2 \text{ or } 25 \times 10^{-3} \text{ N s/m}^2$$

$$\text{Density of acid} = 1840 \text{ kg/m}^3$$

Solution

Reynolds number:

$$Re = \frac{ud\rho}{\mu} = \frac{4G}{\pi\mu d} = \frac{4 \times 1.25}{\pi \times 25 \times 10^{-3} \times 25 \times 10^{-3}} = 2545$$

For a mild steel pipe, suitable for conveying the acid, the roughness e will be between 0.05 and 0.5 mm (0.00005 and 0.0005 m).

The relative roughness is thus: $\dfrac{e}{d} = 0.002 \text{ to } 0.02$

From Figure 3.7: $\dfrac{R}{\rho u^2} = 0.006$ over this range of $\dfrac{e}{d}$

and the velocity is:

$$u = \frac{G}{\rho A} = \frac{1.25}{1840 \times (\pi/4)(0.025)^2} = 1.38 \text{ m/s}$$

The kinetic energy attributable to this velocity will be dissipated when the liquid enters the reservoir. The pressure drop may now be calculated from the energy balance equation and equation 3.19. For turbulent flow of an incompressible fluid:

$$\Delta\frac{u^2}{2} + g\Delta z + v(P_2 - P_1) + 4\frac{R}{\rho u^2}\frac{l}{d}u^2 = 0 \quad \text{(from equation 2.67)}$$

$$\therefore \quad -\Delta P = (P_1 - P_2) = \rho\left[0.5 + 4\frac{R}{\rho u^2}\frac{l}{d}\right]u^2 + g\Delta z$$

$$= 1840\left\{\left[0.5 + 4 \times 0.006\frac{30}{0.025}\right]1.38^2 + 9.81 \times 12\right\}$$

$$= 3.19 \times 10^5 \text{ N/m}^2$$

or: $\underline{\underline{320 \text{ kN/m}^2}}$

3.3.3. Reynolds number and shear stress

For a fluid flowing through a pipe the momentum per unit cross-sectional area is given by ρu^2. This quantity, which is proportional to the inertia force per unit area, is the force required to counterbalance the momentum flux.

The ratio u/d represents the velocity gradient in the fluid, and thus the group $(\mu u/d)$ is proportional to the shear stress in the fluid, so that $(\rho u^2)/(\mu u/d) = (du\rho/\mu) = Re$ is proportional to the ratio of the inertia forces to the viscous forces. This is an important physical interpretation of the Reynolds number.

In turbulent flow with high values of Re, the inertia forces become predominant and the viscous shear stress becomes correspondingly less important.

In steady streamline flow the direction and velocity of flow at any point remain constant and the shear stress R_y at a point where the velocity gradient at right angles to the direction of flow is du_x/dy and is given, for a Newtonian fluid, by the relation:

$$R_y = -\mu \frac{du_x}{dy} = -\frac{\mu}{\rho} \frac{d(\rho u_x)}{dy} \quad (3.25)$$

which gives the relation between shear stress and momentum per unit volume (ρu_x) (equation 3.3). The negative sign in equation 3.25 indicates that the shear stress on the fluid exerts a retarding force on the faster-moving fluid.

In turbulent motion, the presence of circulating or eddy currents brings about a much-increased exchange of momentum in all three directions of the stream flow, and these eddies are responsible for the random fluctuations in velocity u_E. The high rate of transfer in turbulent flow is accompanied by a much higher shear stress for a given velocity gradient.

Thus:

$$R_y = -\left(\frac{\mu}{\rho} + E\right) \frac{d(\rho u_x)}{dy} \quad (3.26)$$

where E is known as the *eddy kinematic viscosity* of the fluid, which will depend upon the degree of turbulence in the fluid, is not a physical property of the fluid and varies with position.