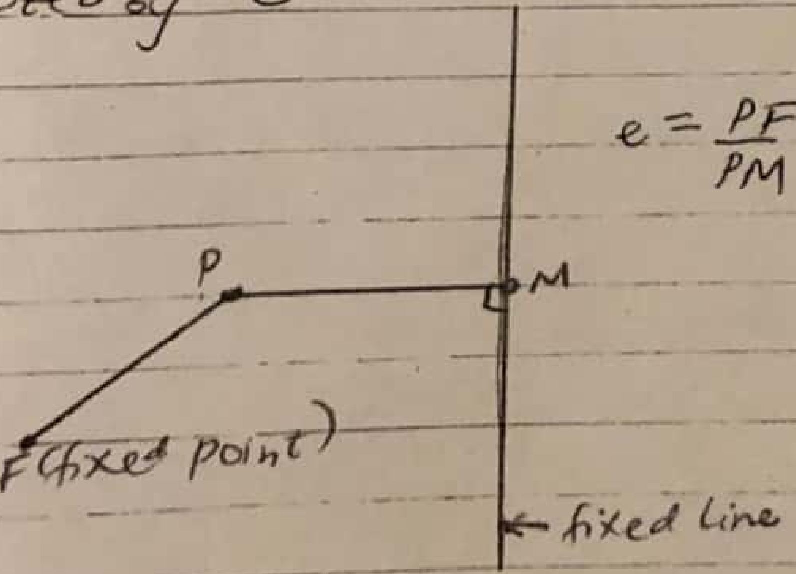
Conic sections

The path of a point which moves so that its distance from a fixed point is in constant ratio to the distance from a fixed line is called the conic sections or a conic. The fixed point "f" is called the focus, the fixed line is called directrix and the constant ratio is called the eccentricity which is denoted by "e".

$$e = \frac{PF}{PM}$$

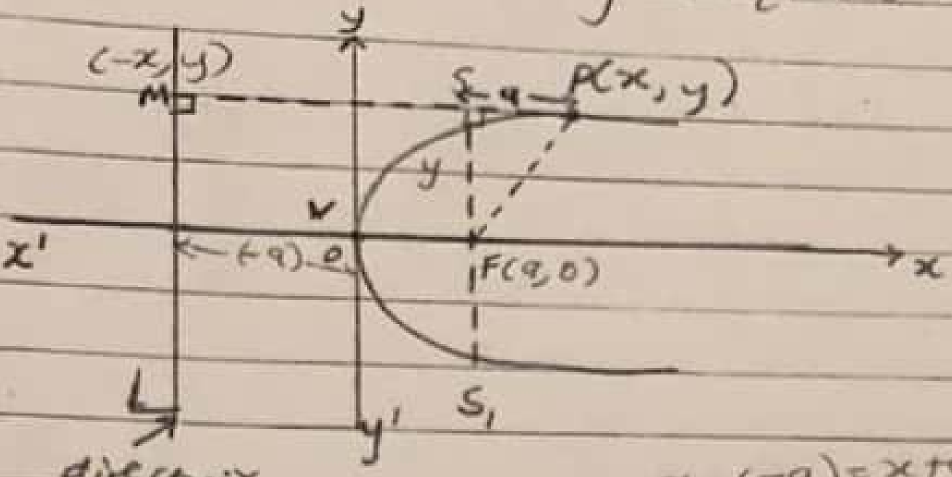


The locus of F is a conic. If $e<1$, the conic is called an ellipse. If $e=1$, it is a parabola. If $e>1$, the conic is a hyperbola.

Parabola.

A parabola is the locus of points in a plane which equidistant from a fixed line and a fixed point.

In parabola, the eccentricity is equal to 1.



$x-(-a)=x+a$

Let L = directrix

F = fixed point

The coordinates of point F is equal to $(a, 0)$. If point P is any point on the parabola such that $\frac{PF}{PM} = e =$ point $(PF)^2 = (x-a)^2 + y^2$ —(I). The distance $PM = x$

But $\frac{PF}{PM} = \frac{1}{1}$

$\Rightarrow PF = PM$

$\Rightarrow PF^2 = PM^2$ —(III)

where $PF = (x-a)^2 + y^2$ —(I)

$PM = x+a$ —(II)

Equation (III) becomes

$(x-a)^2 + y^2 = (x+a)^2$

$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$

Making y^2 the subject...

$y^2 = \cancel{x^2} + 2ax + \cancel{a^2} - \cancel{x^2} + 2ax - \cancel{a^2}$

$y^2 = 4ax$ —— *

$y = \pm\sqrt{4ax}$

$y = \pm 2\sqrt{ax}$

Equation * is the simplest form of the equation parabola.

$\therefore$ The focus $(a, 0) \rightarrow (-\frac{2}{3}, 0)$

Any parabola in its simplest form $y^2 = 4ax$ is that the vertex is at the origin $(0,0)$

$y^2 = 4ax$

$y = mx + c$

$x = -a$

[redacted], $y = 0$

Since the focus is $(a, 0) = (-\frac{2}{3}, 0)$ then the line at point $(a, 0) = (\frac{2}{3}, 0)$ hence by the equation of a line: $y = mx + c$

put $x = \frac{2}{3}[-a]$, $y = 0[0]$, $c = 0$

c is a zero because there is no intercept on the

$y = mx + c$

$0 = m(\frac{2}{3}) + 0$

$\frac{2}{3}m = 0$

then the equation of the line is $x = \frac{2}{3}$.

Equations of tangent and normal on the point (x_1, y_1) the parabola.

The equation of a parabola is $y^2 = 4ax$ so diff the equation with respect to x.

$$y^2 = 4ax$$

$$\frac{2y\frac{dy}{dx}}{2y} = \frac{4a}{2y}$$

$$\therefore \frac{dy}{dx} = \frac{{}^{2}\cancel{4}a}{\cancel{2}y}$$

$$\therefore \frac{dy}{dx} = \frac{2a}{y}$$

$$y' = \frac{2a}{y}$$

where y' represents $\frac{dy}{dx}$.

The gradient of the tangent at the point (x_1, y_1) a parabola is $\frac{2a}{y_1}$. Then, the equation of the tangent is

$(y - y_1) = m(x - x_1)$. $\left[m = \frac{2a}{y_1}\right]$

$(y - y_1) = \frac{2a}{y_1}(x - x_1)$

Expanding...

$yy_1 - y_1^2 = 2ax - 2ax_1$ —— *

Since the point (x_1, y_1) lies on the curve ~~or the~~ (parabola) $y_1^2 = 4ax_1$ then; we have the equation as:

$yy_1 - (4ax_1) = 2ax - 2ax_1$ $[\text{Since } y_1^2 = 4ax_1]$

$yy_1 = 2ax - 2ax_1 + 4ax_1$

$yy_1 = 2ax + 2ax_1$

$yy_1 = 2a(x + x_1)$ —— **

Equation ** is the equation of a tangent to a parabola.

The normal to a curve is at a point is the line passing through the point and perpendicular to the tangent at the point. Therefore, since, the slope of the tangent $= \frac{2a}{y_1}$. Then the slope of the normal $= \frac{-y_1}{2a}$ $\left[\text{since } m_1 = \frac{-1}{m_2}\right]$

Hence the equation of the normal will be equation to...

$y - y_1 = \frac{-y_1}{2a}(x - x_1)$

Examples:

1. Find the equation of the tangent and normal to the parabola $y^2 = 12x$ at the point $(-3, -6)$

solution.

Given that $y^2 = 12x$ —— *

Compare * with $y^2 = 4ax$

$\Rightarrow 4a = 12$

$\therefore a = 3$

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