MESH ANALYSIS

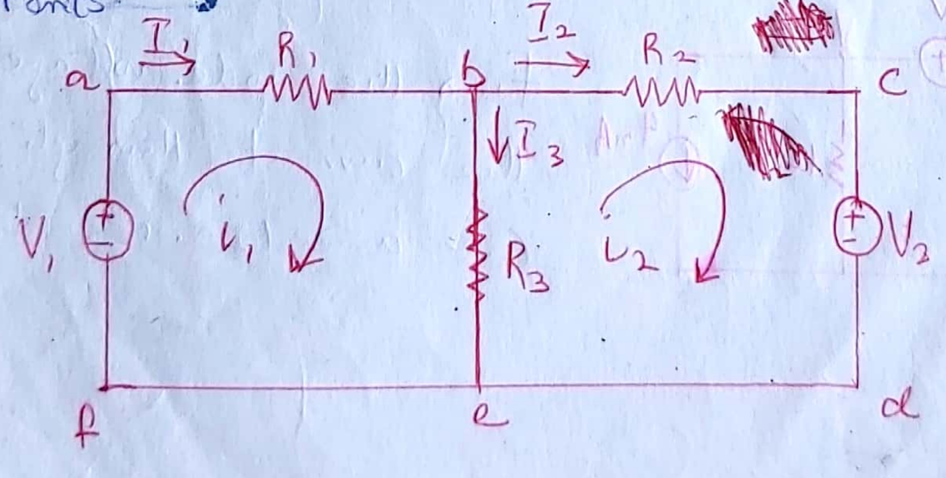
This procedure involves analysing circuits using mesh currents as the circuit variables. This reduces the number of simultaneous equations that need to be solved.

A mesh is a loop that contains no other loop within it; a loop is a closed path with no node passed more than once.

Mesh analysis applies KVL to find unknown currents; it is applicable only to planar circuits — circuits that can be drawn with no branches crossing one another.

To determine mesh currents in mesh analysis:

- Assign mesh currents $i_1, i_2, i_3, \ldots i_n$ to the n meshes.
- Apply KVL to each of the n meshes; use Ohm's law to express the voltages in terms of the mesh currents.
- Solve the resulting n simultaneous equations to get the mesh currents.



i_1, i_2 = mesh currents

I_1, I_2, I_3 = branch currents

$I_1 = i_1$

$I_2 = i_2$

$I_3 = i_1 - i_2$

abefa and bcdeb = meshes

abcdefa $\neq$ mesh

9/ Apply KVL to both meshes:

$$V_1 = i_1R_1 + (i_1 - i_2)R_3$$

$$V_1 = i_1(R_1 + R_3) - i_2R_3$$

and

$$V_2 + i_2R_2 + (i_2 - i_1)R_3 = 0$$

$$V_2 = i_1R_3 - i_2(R_2 + R_3)$$

This can then be solved simultaneously or elimination or cramer's rule.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + R_3 & -R_3 \\ R_3 & -(R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Illustration

Find the branch currents I_1, I_2 and I_3 using mesh analysis in the circuit below:

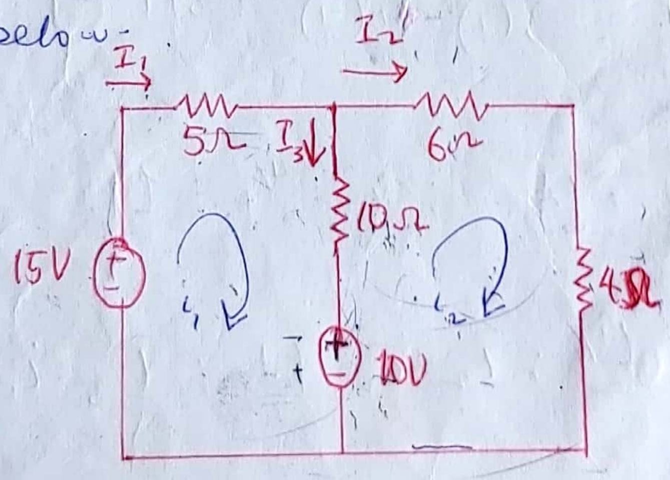


Fig: For Illustration above

Solution

Mesh 1:

$$15 = i_1 5 + (i_1 - i_2)10 + 10$$

$$\Rightarrow 5 = 15i_1 - 10i_2$$

$$1 = 3i_1 - 2i_2 \quad \text{——— i)}$$

15 − 10 =

Mesh 2:

$$10 = (i_2 - i_1)10 + 6i_2 + 4i_2$$

$$\Rightarrow 10 = 20i_2 - 10i_1$$

$$1 = 2i_2 - 1i_1 \quad \text{———} \quad (ii)$$

Using Cramer's rule:

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$i_1 = \Delta_1/\Delta$; $i_2 = \Delta_2/\Delta$

$$\Delta = 3(2) - (-2)(-1) = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 1(2) - 1(-2) = 2+2 = 4$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3(1) - 1(-1) = 3+1 = 4$$

hence $i_1 = 4/4 = \underline{1A}$; $i_2 = 4/4 = \underline{1A}$

and $I_1 = i_1 = \underline{\underline{1A}}$; $I_2 = i_2 = \underline{\underline{1A}}$; $I_3 = i_1 - i_2 = \underline{0}$

or elimination:

$1 = 3i_1 - 2i_2$ — (i)

$1 = 2i_2 - 1i_1$ — (ii)

from (ii) $i_1 = 2i_2 - 1$

put this into (i)

$\Rightarrow 1 = 3(2i_2 - 1) - 2i_2$

$1 = 6i_2 - 3 - 2i_2$

$\Rightarrow 4 = 4i_2$

$i_2 = 4/4 = \underline{1A}$

and from $i_1 = 2i_2 - 1$

$\Rightarrow 1 = 2i_2 - 1$

$2i_2 = 2$

$i_2 = 2/2 = \underline{1A}$

$I_1 = i_1 = \underline{\underline{1A}}$

$I_2 = i_2 = \underline{\underline{1A}}$

$I_3 = i_1 - i_2 = 1 - 1 = \underline{0}$

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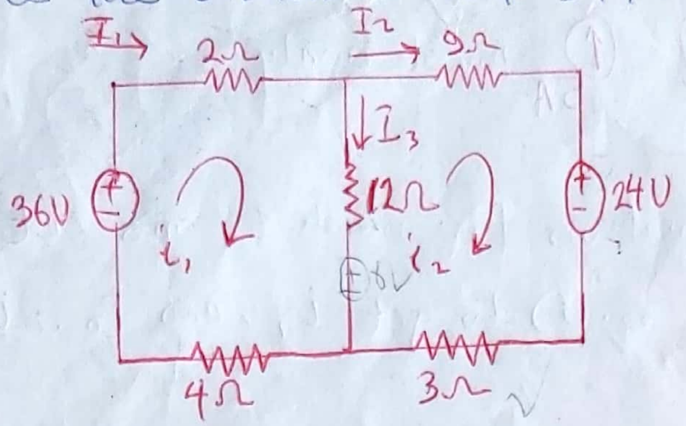
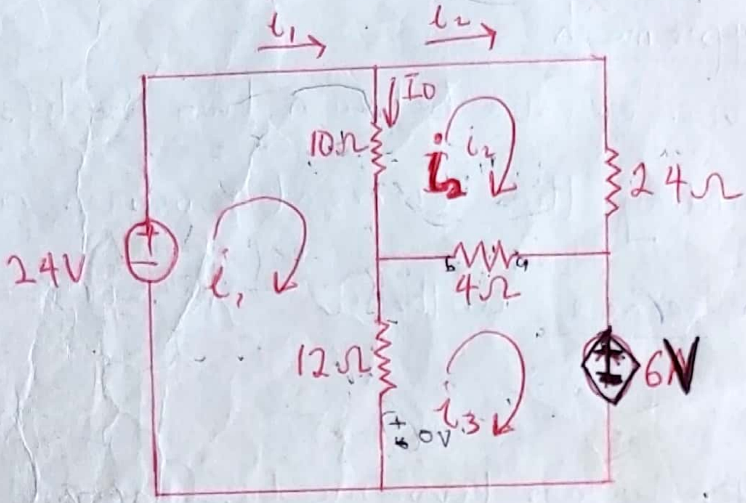
Classwork | Practice Questions

(i) Calculate the branch currents I_1, I_2 and I_3 in the circuit below.



(ii)



Ans:

$i_1 = 2.25A$

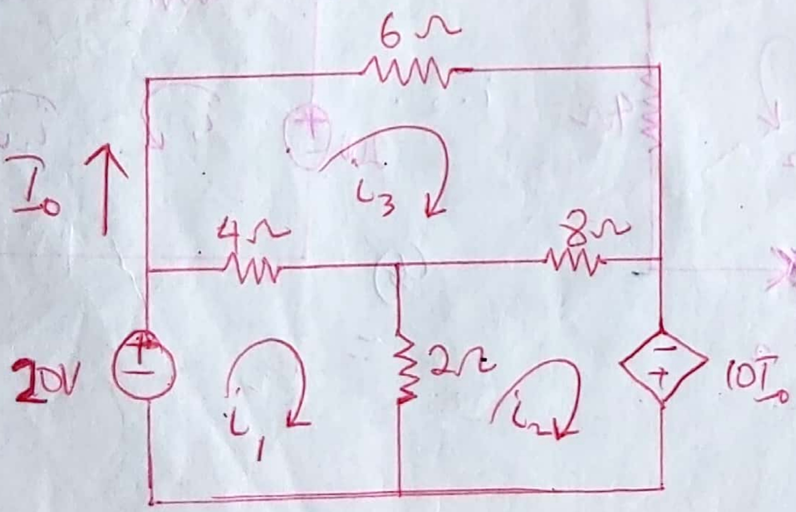
$i_2 = 0.75A$

$i_3 = 1.5A$

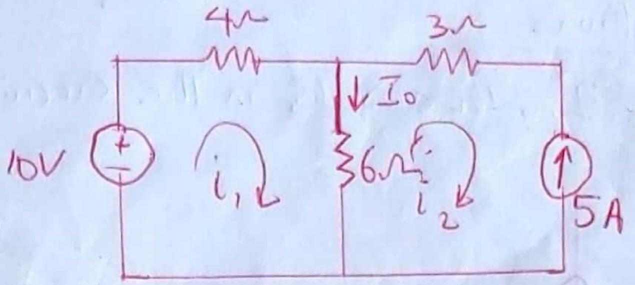
$I_0 = 1.5A$

Use mesh analysis to obtain the current I_0 in the circuit above.

(iii) Use mesh analysis to obtain I_0 in the circuit below.



Ans: $-5A$



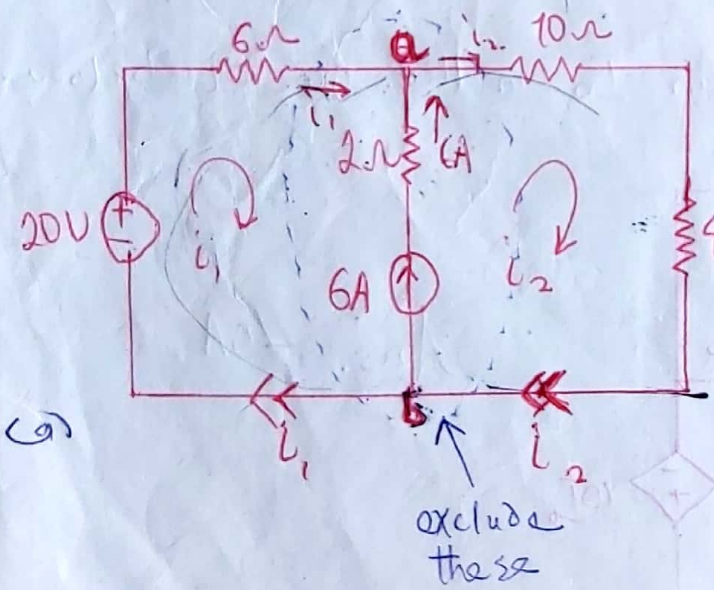
Find I_o in this circuit, using mesh analysis.

Hint: Write a mesh equation for mesh 1, and set $i_2 = -5A$.
Substitute this into eq. (i) to get i_1, and $I_o = i_1 - i_2$

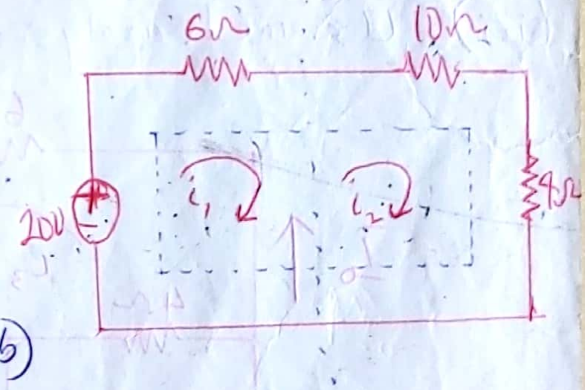
Special Case: Supermesh

When a current source exists between two meshes, this is approached by excluding the current source and any element(s) connected in series with it — this results in a supermesh.

A supermesh exists when two meshes share a current source — dependent or independent — in common.



i.e.



$i_1 + 6 = i$

Figs (a); (b): Two meshes with a common current source; Supermesh created by excluding (6A & 2Ω) elements

$i_1 + 6 = i_2$

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The supermesh in (b) forms the periphery of the two meshes, and is treated separately. A circuit consisting of two (or more) intersecting supermeshes should be resolved by combining these supermeshes to form a larger supermesh.

Supermeshes are treated differently because KVL used in mesh analysis requires a knowledge of the voltage across each branch, but the voltage across a current source is not known in advance. They must however, satisfy KVL like any other mesh.

Applying KVL to the supermesh in (b):

$$20 = 6i_1 + 10i_2 + 4i_2$$

$$20 = 6i_1 + 14i_2 \quad \text{——— (i)}$$

Applying KCL to any node (a or b) where the two meshes intersect:

i_1 and 6A flows into node a and out of node b

i_2 flows out of node a and enters node b

hence,

$$i_1 + 6 = i_2 \quad \text{——— (ii)}$$

Substituting (ii) into (i)

$$\Rightarrow \quad 20 = 6i_1 + 14(i_1 + 6)$$

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$$20 = 6i_1 + 14i_1 + 84$$

$$-64 = 20i_1$$

$$i_1 = -64/_{20} = \underline{\underline{-3.2A}}$$

and $i_2 = i_1 + 6 = -3.2 + 6 = 2.8$

$$i_2 = \underline{\underline{2.8A}}$$

Conclusively, in solving circuits with supermesh(es), the following need to be known:

(i) The current source in the supermesh provides the constraint equation necessary to solve for the mesh currents.

(ii) A supermesh has no currents of its own.

(iii) A supermesh requires the application of both KVL & KCL.

Example:

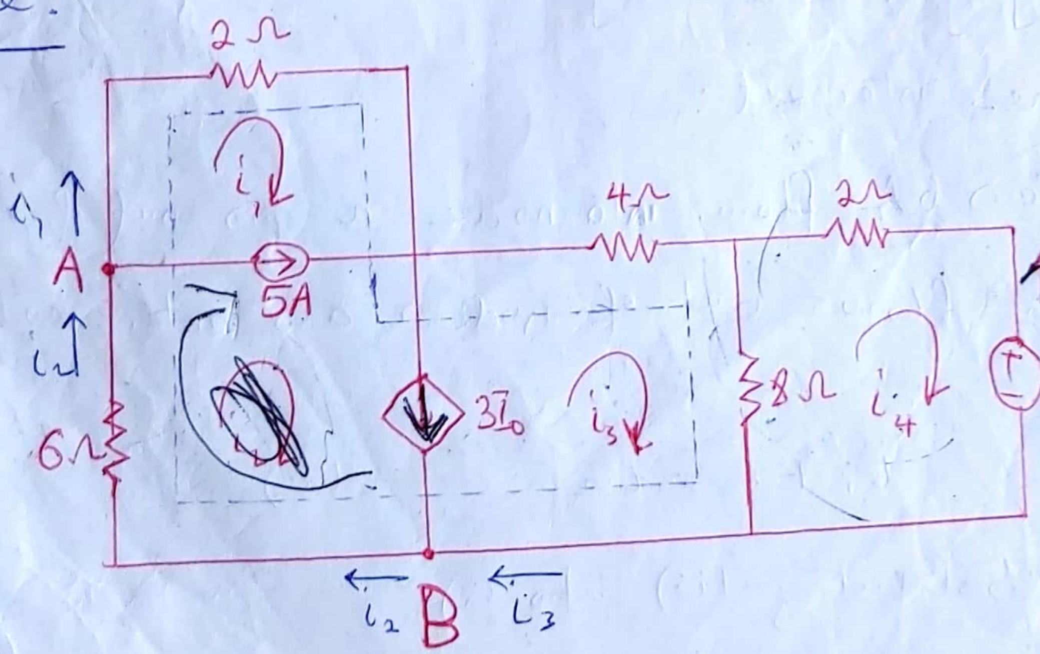

Fig: For mesh analysis example.

12/ Find i_1, i_2, i_3 and i_4 using mesh analysis for the circuit shown.

__Analysis:__

- Meshes 1 and 2 form a supermesh since they have an independent current source (5A) in common.
- Meshes 2 and 3 also form another supermesh because they have a dependent current source ($3I_o$) in common.
- These two supermeshes intersect to form a larger supermesh.

Applying KVL to the larger supermesh:

$$\cancel{2i_1 + 4i_3 =}$$

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$2i_1 + 12i_3 + 6i_2 - 8i_4 = 0$$

$$i_1 + 6i_3 + 3i_2 - 4i_4 = 0 \quad \text{—— i)}$$

For the 5A independent current source, KCL is applied at node A:

$$i_2 = i_1 + 5 \quad \text{—— ii)}$$

For the dependent current source $3I_o$, KCL is applied at node B:

$$i_2 = i_3 + 3I_o$$

but $I_0 = -i_4$

$\Rightarrow i_2 = i_3 - 3i_4$ —— (iii)

Applying KVL to mesh 4 !

$10 + 2i_4 + 8i_4 - 8i_3 = 0$

$10 = -10i_4 + 8i_3$

$5 = -5i_4 + 4i_3$ —— (iv)

~~substitution (iv) into (iii) i.e.~~

$\cancel{10i_4 = 4i_3 - 5}$

$\cancel{i_4 = \frac{4i_3 - 5}{10}}$ ~~eq. (iii)~~ ~~eq. (ii)~~

$\cancel{\Rightarrow i_2 = i_3 - \frac{3(4i_3 - 5)}{10} = i_1 + 5}$

$\cancel{\Rightarrow 10i_3 - 3(4i_3 - 5) = 10i_1 + 50}$

$\cancel{10i_1 = 10i_3 - 12i_3 + 15 - 50}$

$\cancel{10i_1 = -2i_3 - 35}$

So we have 4 equations - 4 unknows i.e.

$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$ —— (i)

$i_1 - i_2 = -5$ —— (ii)

$i_2 - i_3 + 3i_4 = 0$ —— (iii)

$4i_3 - 5i_4 = 5$ —— (iv)

From (ii) $i_1 = i_2 - 5$

put this into (i) :

$$i_2 - 5 + 3i_2 + 6i_3 - 4i_4 = 0$$

$$4i_2 + 6i_3 - 4i_4 = 5 \quad \text{—— (iv)}$$

From (iii), $i_2 = i_3 - 3i_4$

Put this into (iv):

$$4(i_3 - 3i_4) + 6i_3 - 4i_4 = 5$$

$$4i_3 - 12i_4 + 6i_3 - 4i_4 = 5$$

$$10i_3 - 16i_4 = 5 \quad \text{—— (vi)}$$

From (v), $i_3 = \dfrac{5 + 5i_4}{4}$

Put this into (vi):

$$10\left(\frac{5+5i_4}{4}\right) - 16i_4 = 5$$

$$12.5 + 12.5i_4 - 16i_4 = 5$$

$$12.5i_4 - 16i_4 = -7.5$$

$$-3.5i_4 = -7.5$$

$$i_4 = \frac{-7.5}{-3.5} = 2.142865$$

$$i_4 \simeq \underline{\underline{2.143\,A}}$$

Substituting this into eq. (v):

$$4i_3 - 5(2.142865) = 5$$

$$4i_3 = 5 + 10.714325 = 15.714325$$

$i_3 = \frac{15.714325}{4} = 3.92858$

$\therefore i_3 \simeq \underline{3.930A}$

From (iii), $i_2 = i_3 - 3i_4 = 3.93 - 3(2.143)$

$i_2 = 3.93 - 6.429 = -2.499$

$\therefore i_2 \simeq \underline{-2.50A}$

From (ii), $i_1 = i_2 - 5$

$\Rightarrow i_1 = -2.50 - 5 = -7.50$

$\therefore i_1 = \underline{-7.50A}$

Exercise

Use mesh analysis to determine i_1, i_2 and i_3 in the circuit below.

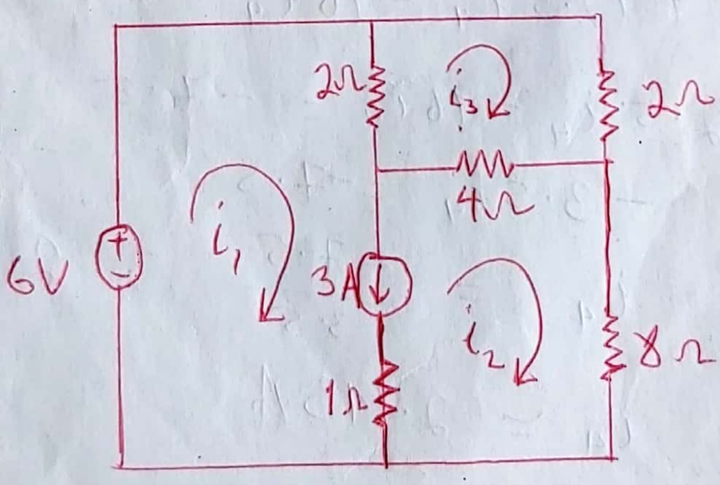

fig: for exercise above

Ans: $i_1 = 3.474A$; $i_2 = 0.4737A$

$i_3 = 1.1052A$