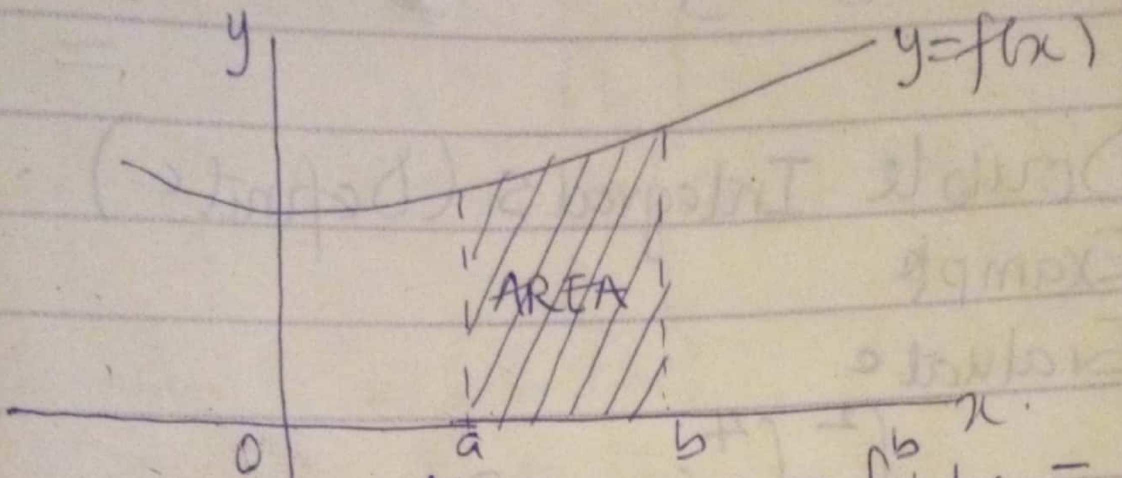
Add this to the Note given in class last week.

Area Under a Curve

We shall assume that the eqn of d curve under consideration is given by $y = f(x)$. We shall find the area under curve $y = f(x)$ at $x = a$ and $x = b$. This is shown below



NOTE $A = \int_a^b y\,dx = \int_a^b f(x)\,dx$

Example

Find the area bounded by the curves between

$y = x^2 + 2x + 1$, ~~[illegible]~~

~~at~~ $x = 1$ and $x = 2$

$$A = \int_a^b y\,dx$$

Soln

$$A = \int_1^2 \overset{y\,dx}{f(x)\,dx} = \int_1^2 (x^2 + 2x + 1)\,dx$$

$$= \left[\frac{x^3}{3} + \frac{2x^2}{2} + x\right]_1^2$$

$$\left[\frac{2^3}{3} + \frac{2(2)^2}{2} + 2\right] - \left[\frac{1^3}{3} + \frac{2(1)^2}{2} + 1\right]$$

$$= \left[\frac{8}{3} + \frac{6}{1}\right] - \left[\frac{1}{3} + \frac{2}{1}\right]$$

$$\left[\frac{8+18}{3}\right] - \left[\frac{1+6}{3}\right]$$

$$\frac{26}{3} - \frac{7}{3} = \frac{19}{3} \text{ square units}$$

$$=$$

AREA UNDER A CURVE IN PARAMETRIC FORM

It is possible to find the area under a given curve which is represented by a parametric eqn.

Exple

1. Find the area under the curve with a parametric eqn $x = at^2$, $y = 2at$ at $t = 1$ & $t =$

Soln

Let A represent the area, then

$A = \int_a^b y\,dx$ where a & b are d limit values of d varia

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Given $y = 2at$, then

$A = \int_a^b 2at\, dx$

Given $x = at^2$

$\frac{dx}{dt} = 2at$

$dx = 2at\,dt$

$A = \int_1^2 2at\,(2at\,dt)$

$= \int_1^2 4a^2t^2\,dt \quad = 4a^2\int_1^2 t^2\,dt$

$= \left[\frac{4a^2t^3}{3}\right]_1^2$

$= \left[\frac{4a^2 2^3}{3}\right] - \left[\frac{4a^2 1^3}{3}\right]$

$= \frac{32a^2}{3} - \frac{4a^2}{3}$

$= \frac{28a^2}{3}$ square units

Exple 2

Given a parametric eqns $x = a\sin\theta$, $y = b\cos\theta$, find the area under the curve between $\theta = 0$ and $\theta = \pi$.

Sol^n

Let A be the area, then

$$A = \int_a^b y\,dx$$

$$A = \int_0^{\pi} (b\cos\theta)\,dx$$

Given $x = a\sin\theta$

$$\frac{dx}{d\theta} = a\cos\theta \Rightarrow dx = a\cos\theta\,d\theta$$

Thus

$$A = \int_0^{\pi} (b\cos\theta)(a\cos\theta)\,d\theta$$

$$A = \int_0^{\pi} (ab\cos^2\theta)\,d\theta$$

$$= ab\int_0^{\pi} \cos^2\theta\,d\theta$$

Since $\int \cos^2\theta\,d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4}$

It implies

$$ab\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$= ab\left[\frac{\pi}{2} + \frac{\sin 2\pi}{4} - \left(\frac{0}{2} + \frac{\sin 0}{4}\right)\right]$$

$$A = ab\left[\frac{\pi}{2} + 0\right] = \frac{\pi ab}{2} \text{ square units.}$$

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