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CONTROL SYSTEMS ENGINEERING (BIOMEDICAL APPLICATIONS)

Blood pressure control during anaesthesia

> The objectives of anaesthesia are to eliminate pain, awareness, and natural reflexes so that surgery can be conducted safely. In a modern operating room, the depth of anaesthesia is the responsibility of the anaesthetist.

- Vital parameters, such as blood pressure, heart rate, temperature, blood oxygenation, and exhaled **carbon dioxide**, are controlled within acceptable bounds by the anaesthetist; while ensuring that adequate anaesthesia is maintained during the entire surgical procedure.
- Automating functions that are amenable to automatic control frees the anaesthetist to attend to other functions not easily automated. Our control goal then is to develop an automated system to regulate the depth of anaesthesia (measurable via the mean arterial pressure MAP).

SYSTEM DESIGN SUMMARY

» Establish the control goals: Regulate the mean arterial pressure (MAP) to any given set point and to thus control the depth of anaesthesia

» Identify the variable to be controlled: Mean arterial pressure

» Write the specifications: e.g. Settling time; percent overshoot; tracking error; disturbance rejection; system sensitivity

» Establish system configuration: e.g. Determine system architecture, representable as block diagrams.

» Obtain models for the elements of the system i.e. the process, the actuator, and sensor: Determine appropriate model: say via conservation laws and constitutive relations

» Design a controller, analyse the performance, and optimise the system: "Many" methods and paradigms available

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Example 4.3: Blood pressure control during anaesthesia

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— Control Goal: Regulate the depth of anaesthesia. ~~The level~~

- Based on clinical experience and the procedures followed by the anaesthetist, we determine the variable to be controlled as the mean arterial pressure. (The level of the MAP serves as a guide for the delivery of inhaled anaesthesia.

Variable to be controlled

- Hence the control goal can be rewritten as: to regulate the mean arterial pressure to any desired set-point and maintain the prescribed set-point in the presence of unwanted disturbances

The control goal

— Control Design Specifications

- Some considerations:
  - » The control system should respond rapidly enough and smoothly to changes in the MAP set-point (made by the anaesthetist) without excessive overshoot ~~and set-point~~
  - » The MAP must attain ~~the~~ desired set-point values
  - » The closed loop system should minimise the effects of unwanted disturbances:
    - → Surgical disturbances :- e.g. a skin incision can increase the MAP rapidly by 10 mmHg
    - → Measurement errors :- e.g. calibration errors and effect of random/stochastic noise.
  - » The same control system will be applied to different patients and we cannot (for practical reasons) have a separate model for each patient. We must therefore have a closed-loop system that is insensitive to changes in the process parameters ~~environment~~
- In explicit terms, and based on clinical experience, the control specifications can be stated as follows:
  - → DS1: Settling time less than 20 mins for a 10% step change from the MAP set-point.

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→ DS2: Percent overshoot less than 15% for a 10% step change from the MAP set-point

→ DS3: Zero steady-state tracking error to a step change from the MAP set-point

→ DS4: Zero steady-state error to a step surgical disturbance input (of magnitude $|d(t)| \leq 50$) with a maximum response less than ±5% of the MAP set-point

→ DS5: Minimum sensitivity to process parameter changes.

— System Configuration and Modelling

- The major system elements identified: controller, anaesthesia pump/vapouriser, sensor and patient.

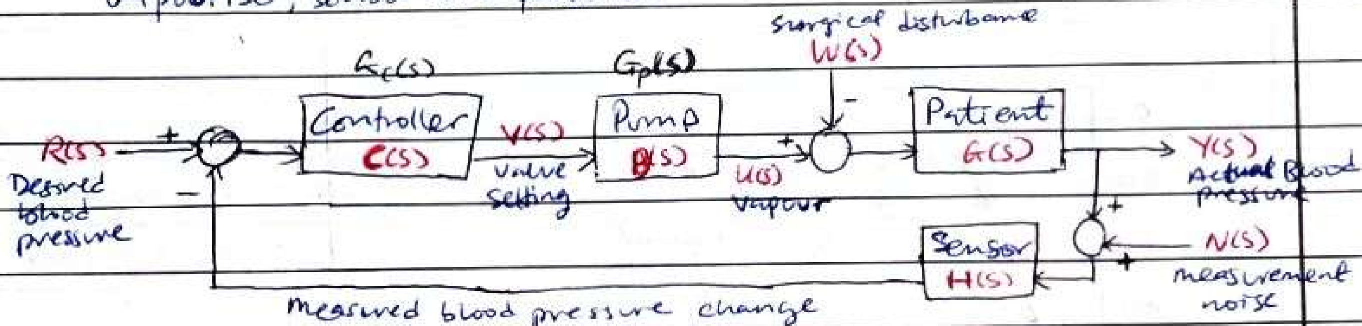

fig Blood pressure control system configuration

- Pump/Vapouriser model: A simple model could be based on setting the rate of change of the output vapor equal to the input valve setting (decided by the controller):

$$\dot{u}(t) = V(t)$$

$$s u(s) = V(s)$$

$$\frac{u(s)}{V(s)} = \frac{1}{s} = G_p(s)$$

with an impulse response $u(t) = 1, \ t \geq 0$

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• Patient model: A model based on the knowledge of the under-lying physical processes would, in general, be nonlinear, time-varying, multi-input, and multi-output. In order to obtain a linear approximation so that linear control systems analysis and design can be applied, we would want to restrict ourselves to small changes in blood pressure from a given set point (such as 100 mmHg). This is based on the assumption that in a small region around the set-point the patient behaves in a linear time-invariant fashion.

This approach is useful because the objective is regulation of the blood pressure around a given base line (set-point)

Obtain a linear approximation via the impulse response of the system has been successfully used in the past.

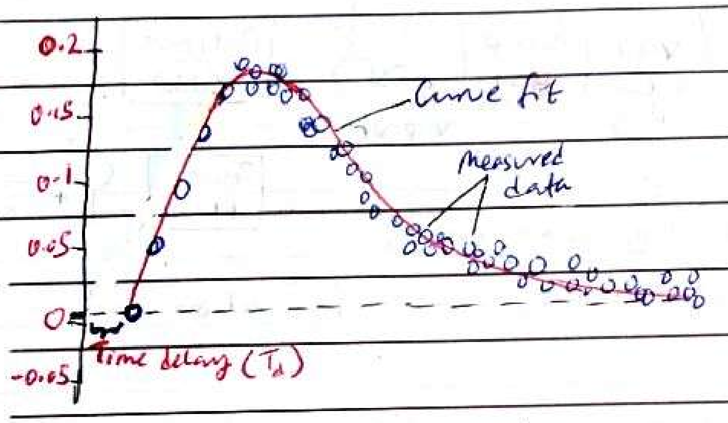

fig. Mean arterial pressure (MAP) impulse response for a hypothetical patient.

— Notice that the impulse response initially has a time delay (T_d). This reflects that it takes a finite amount of time for the patient MAP to respond to the infusion of anaesthesia vapour.

— Ignoring the time delay, a reasonable fit for the data in the above plot is given by

impulse response

$$y(t) = te^{-pt} \quad t \geq T_d$$

» we assume that $y(t) = 0$ for $0 \leq t < T_d$

» different patients are associated with different values of 'p'.

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→ The corresponding transfer function to the impulse response is

$$R(s) = \frac{1}{(s+p)^2}$$

- Sensor model: Assuming a perfect noise-free and unmodified measurement, the sensor model could be

$$H(s) = 1$$

thus yielding a unity feedback system.

— Controller Design ~~transmission~~

> To achieve as closely as possible the system performance specification

> A popular and intuitive controller model is the PID (Proportional-Integral-Derivative) controller:

$$G_c(s) = k_p + s.k_D + \frac{k_I}{s}$$

$$= \frac{k_D s^2 + k_p s + k_I}{s}$$

where k_p, k_D, and k_I are the controller gains to be determined to satisfy all design specifications

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>> Choosing k_p, k_D, and k_I

- The steady state tracking error, assuming that $T_d(s) = N(s) = 0$ is

$$E(s) = R(s) - Y(s) = R(s)(1 - G_{CL}(s)) \quad \text{for} \quad \frac{Y(s)}{R(s)} = G_{CL}(s)$$

since $$G_{CL}(s) = \frac{G_c(s) G_p(s) G(s)}{1 + G_c(s) G_p(s) G(s) H(s)}$$

$$E(s) = \frac{1}{1 + G_c(s) G_p(s) G(s)} R(s)$$

$$= \frac{s^4 + 2ps^3 + p^2 s^2}{s^4 + 2ps^3 + (p^2 + k_D)s^2 + k_p s + k_I} R(s)$$

Using the final-value theorem, we determine the steady-state tracking error to be

$$\lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{R_0 (s^4 + 2ps^3 + p^2 s^2)}{s^4 + 2ps^3 + (p^2 + k_D)s^2 + k_p s + k_I} = 0$$

where $R(s) = R_0/s$ is a step input of magnitude R_0.

$$\lim_{t \to 0} e(t) = \lim_{s \to 0} sE(s) = 0$$

- The design specification DS 3 is thus satisfied by the given PID controller, and for any non zero values of k_p, k_D, k_I.
  - ∴ The integral term, $\frac{k_I}{s}$, in the PID controller is the reason that the steady-state error to a unit step is zero

- For the effect of a step disturbance input, let $R(s) = N(s) = 0$
  - ∴ We want the steady-state output $Y(s)$ to be zero for a step disturbance. That is, we want the effect of a step disturbance on the output to be eliminated over time.

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Disturbance transfer function

~~[illegible]~~, The transfer function from disturbance T_d to the output $Y(s)$ ~~[illegible]~~ is determined thus:

- Observe that $Y = (-T_d + U)G = -T_d G + UG$

$U = V G_p$

$V = (R - J) G_c$

where J is defined to be the output of $H(s)$

$J = (N + Y) H$

$\therefore V = [R - (N+Y)H] G_c$

$U = [R - (N+Y)H] G_p G_c$

~~[illegible]~~

$\Rightarrow Y = -T_d G + [R - (N+Y)H] G_p G_c G$

for our consideration of the effect of T_d only on the output of the system, let $R = N = 0$, (recall $H=1$),

$\Rightarrow Y = -T_d G + [0 - (0+Y)] G_p G_c G$

$= -T_d G - Y G_p G_c G$

$\therefore Y + Y G_p G_c G = -T_d G$

$Y(1 + G_p G_c G) = -T_d G$

$$\therefore Y(s) = \frac{-T_d G(s)}{1 + G_p(s) G_c(s) G(s)}$$

Hence for $G_c(s) = \dfrac{k_D s^2 + k_p s + k_I}{s}$

$G(s) = \dfrac{1}{(s+p)^2}$

$G_p(s) = \dfrac{1}{s}$

$$Y(s) = \frac{-s^2 T_d}{s^4 + 2ps^3 + (p^2 + k_D)s^2 + k_p s + k_I}$$

when $T_d = \dfrac{D_o}{s}$.

we find that

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{-D_o s^2}{s^4 + 2p^3 + (p^2 + k_D)s^2 + k_p s + k_I} = 0$$

Thus a step disturbance input of magnitude D_o will produce no ~~change~~ output in the steady state as desired (DS 4)

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* For the part of DS4 that requires a transient part to the effect of T_d of no more than ± 5% of MAP, we will determine that by trial-and-error during simulation of the controlled system.

• Sensitivity analysis

Sensitivity refers to how a closed loop transfer function changes for a change in ~~some~~ element of the closed loop. The sensitivity of a closed loop system is of prime importance

.. System sensitivity is defined as the ratio of the percentage change in the closed loop system transfer function to the change of a block/process transfer function (or parameter) for a small incremental change..

The sensitivity of the closed loop transfer function to changes in 'p' is given by

$$S_p^T = S_G^T \, S_p^G$$ (observe the chain rule)

Recall that $G = \frac{1}{(s+p)^2}$.

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} \quad \text{and} \quad S_p^G = \frac{\partial G}{\partial p} \cdot \frac{p}{G}$$

$$S_p^G = \frac{-2p}{s+p}$$

If T is the closed loop transfer function, then $S = 1 - T \equiv S_G^T$

$$\therefore \quad S_G^T = \frac{1}{1 + G_c G_p G}$$

$$= \frac{2p(s+p)s^2}{s^4 + 2ps^3 + (p^2 + k_D)s^2 + k_p s + k_I}$$

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We must evaluate the sensitivity function, S^T_p, at various values of frequency. Because, ~~[illegible]~~ we desire slow changes of the MAP, this coincides with operations at very low frequencies. Hence for low ~~freq~~ frequencies we can approximate the system sensitivity to p, S^T_p, by

$$S^T_p \approx \frac{2p^2 s^2}{k_I}$$

So at low frequencies and for a given "p", we can reduce the system sensitivity to variations in p by increasing the PID gain, k_I.

HOMEWORK:

Using simulations on Matlab/Scilab/Octave, experimentally (i.e. by trial-and-error, in this case) determine k_P, k_D and k_I gains that satisfy all the design specifications (DS1–5)

- Choose a value of P between 1 and 5.

Insulin Delivery Control System

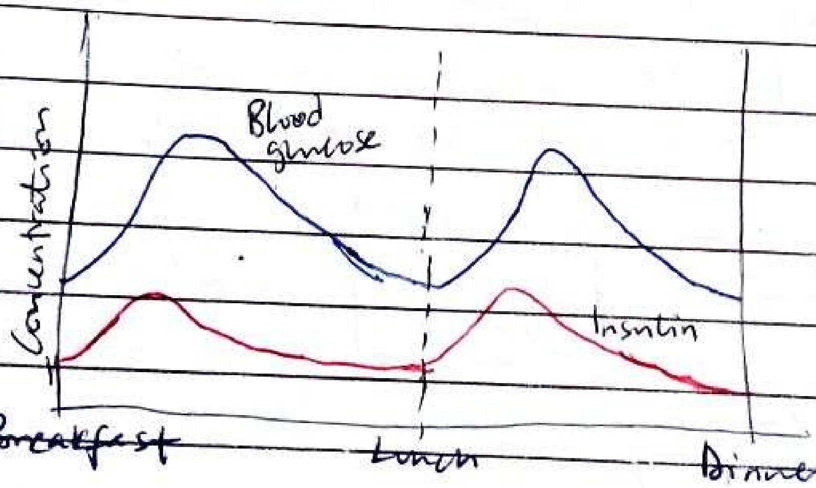

fig: The blood glucose and insulin levels for a healthy person

> Control Goal: Design a system to regulate the blood glucose concentration of a diabetic by controlled dispensing of insulin.

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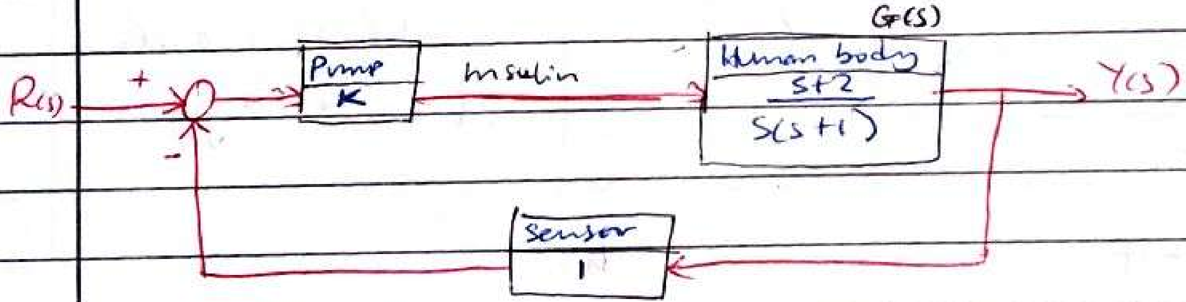
> Variable to be controlled: Blood glucose concentration
> Control Design Specification: Provide a blood glucose level for the diabetic that closely approximates (tracks) the glucose level of a healthy person.
  - The above is a qualitative, hence, imprecise, specification. For it to be useful, a quantitative representation is required i.e. what does it mean, in numbers/figures of a specified quantity to track the normal blood glucose level of a person.
  - We have previously seen steady-state and transient response specifications.
> System Design to Achieve Specification: ~~Select~~ Choose system configuration and components and design ~~appropriate~~ controller.
> Test and Evaluate System: Simulation, HIL (Hardware-in-the-loop), CIL (Controller-in-the-loop), real system testing and evaluation, animal and human trials...

Example Automatically controlled insulin injection by means of a pump and a sensor that measures blood sugar can be very effective for the management of diabetes.

Given the blood sugar level control system below.



Calculate the suitable gain 'k' so that the percentage overshoot of the step response due to the drug injection is P.O. = 7%.

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$R(s)$ is the desired blood-sugar level; $Y(s)$ is the actual blood-sugar level

- Exercise: Use trial-and-error methods, via simulation, to find a suitable 'k'
- The standard second order system has the form

$$G_1(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

the resulting closed loop system is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{kG(s)}{1+kG(s)}$$

$$= \frac{k(s+2)}{s^2+s(k+1)+2k}$$

observe DC gain $=1$
a zero at $s=-2$

$$= \left(\frac{k}{s^2+s(k+1)+2k}\right)\left(\frac{s+2}{1}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{2k}{s^2+s(k+1)+2k}\right)\left(\frac{s+2}{1}\right) = \left(\frac{2k}{s^2+s(k+1)+2k}\right)\left(\frac{s}{2}+1\right)$$

¿What is the effect of a zero on a nominal system?

Say $(s+2)G_1(s) = Y_2(s)/R(s) = G_2(s)$

or $\left(\frac{s}{2}+1\right)G_1(s) = Y_2(s)/R(s) = G_2(s)$ let's use this formulation.

where $G_1(s)$ is the nominal system without any zeros.

$$G_2(s)R(s) = Y_2(s) = R(s)\left(\frac{s}{2}+1\right)G_1(s)$$ (response with zero)

$$= R(s)G_1(s)\left(\frac{s}{2}\right) + R(s)G_1(s)$$

But $R(s)G_1(s) = Y_1(s)$ (the response when no zero is present.)

$$Y_2(s) = \left(\frac{s}{2}\right)Y_1(s) + Y_1(s)$$

$$y_2(t) = \frac{1}{2}\dot{y}_1(t) + y_1(t)$$

⇒ Hence, the larger the value of z, the lower the effect of the zero on the nominal response and vice versa.

⇒ The effect of the zero is also lower when the the response is slowly changing. Obviously there is no effect where $y_1(t)$ is constant.

see other note

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>> In general, rise time is decreased and overshoot increased by a left-half-plane zero

>> The effect of adding a righ-half-plane zero is to increase rise time and induce undershoot.

- Percent overshoot for the standard first order system:

$P.O. = 100 \exp(-\xi\pi/\sqrt{1-\xi^2})$

This should lead to $\xi \approx 0.65$ for 7% overshoot

Check, words

The zero given/obtained is ~~s=~~ $s=-2$ i.e. a left-half-plane zero

— Intuitively, we'll need a K that leads to **more** damping (of the nominal system with no zeros) to counter the effect of the zero

* Methods exist to more analytically/constructively find the K, but we will not cover it.