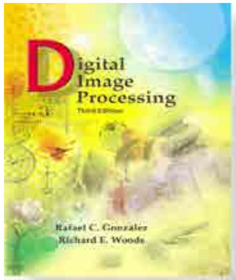


Chapter 6

Image Restoration and Reconstruction

Introduction

- Image enhancement : *subjective* process
- Image restoration : *objective* process

- *Restoration*: recover an image that has been degraded by using a priori knowledge of the degradation phenomenon

- *Process*: modelling the degradation and applying the inverse process to recover the original image
e.g.: “de-blurring”

Some techniques are best formulated in the spatial domain (e.g. additive noise only), others in the frequency domain (e.g. de-blurring)

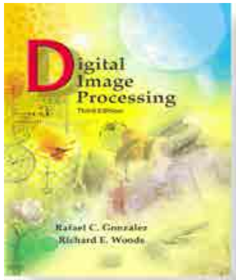
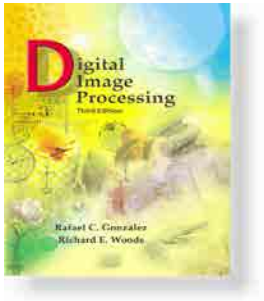


Image Restoration and Reconstruction

1. A Model of the Image Degradation/Restoration Process
2. Noise Models
3. Restoration in the Presence of Noise Only - Spatial Filtering
4. Periodic Noise Reduction by Frequency Domain Filtering
5. Estimating the Degradation Function
6. Inverse Filtering
7. Minimum Mean Square Error (Wiener) Filtering
8. Geometric Mean Filter



1. A Model of the Image Degradation/Restoration Process

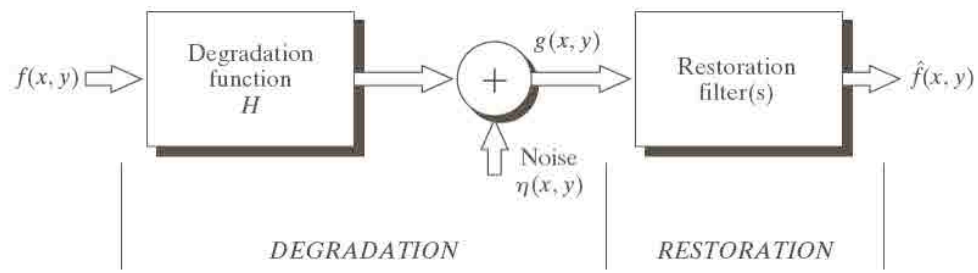


FIGURE 5.1 A model of the image degradation/restoration process.

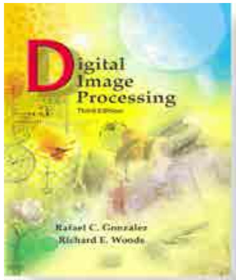
If H is a linear, position-invariant process

Spatial domain representation of the degraded image:

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

Frequency domain representation:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$



2. Noise Models

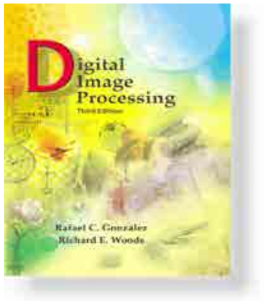
Principal source of noise during image acquisition and/or transmission
Example of factors affecting the performance of imaging sensors:

- Environment conditions during acquisition (e.g: light levels and sensor temperature)
- Quality of the sensing elements

2.1 Spatial and Frequency Properties of Noise

Noise will be assumed to be:

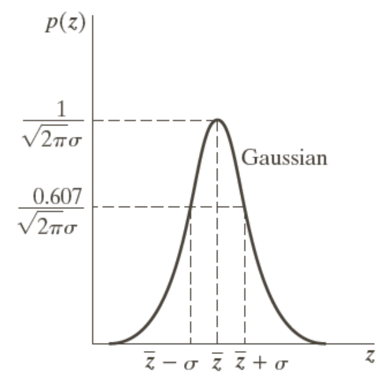
- independent of spatial coordinates (except the spatially periodic noise of 2.3)
- uncorrelated w.r.t. the image (i.e. no correlation between pixel values and the values of noise components)



2. Noise Models

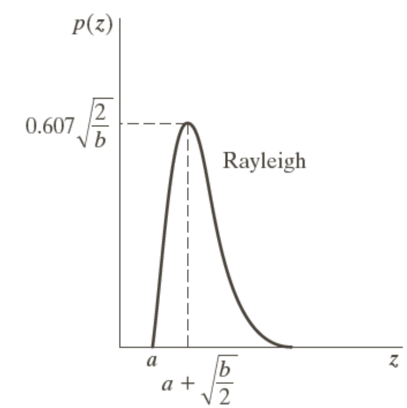
Spatial noise descriptor: statistical behaviour of the intensity values in the noise component => Random variables characterized by a Probability Density Function (PDF)

2.2 Some Important Noise Probability Density Functions



<u>Gaussian (*Normal*) Noise</u>

PDF of a Gaussian random variable z: $$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$



<u>Rayleigh Noise</u>

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Mean: $\bar{z} = a + \sqrt{\pi b/4}$ Variance: $\sigma^2 = \frac{b(4-\pi)}{4}$

2. Noise Models

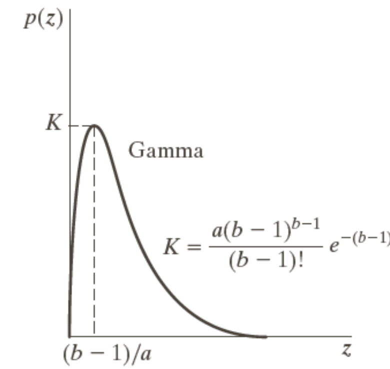
2.2 Some Important Noise Probability Density Functions

<u>Erlang (Gamma) Noise</u>

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$a > 0$

b positive integer



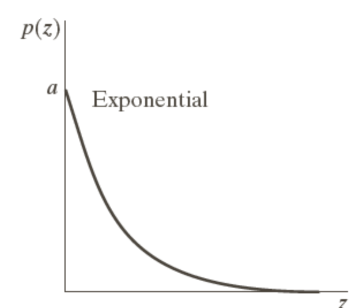
Mean: $\bar{z} = \dfrac{b}{a}$ Variance: $\sigma^2 = \dfrac{b}{a^2}$

<u>Exponential Noise</u>

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

(cf. Erlang noise with $b = 1$)

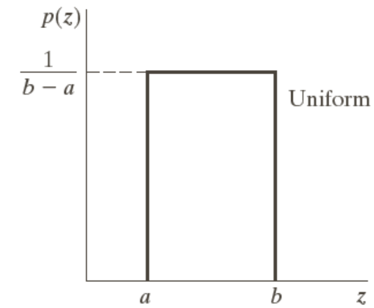
$a > 0$



2.2 Some Important Noise Probability Density Functions

<u>Uniform Noise</u>

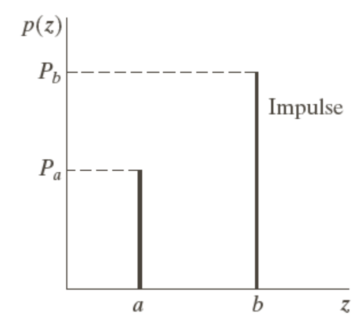
$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$



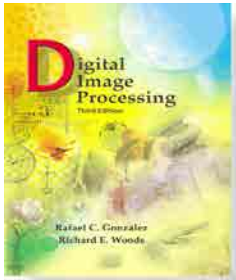
$$\text{Mean: } \bar{z} = \frac{a+b}{2} \qquad \text{Variance: } \sigma^2 = \frac{(b-a)^2}{12}$$

<u>*Bipolar* impulse noise (salt-and-pepper)</u>

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$



$P_a = P_b \Rightarrow$ *unipolar* noise



2.2 Some Important Noise Probability Density Functions

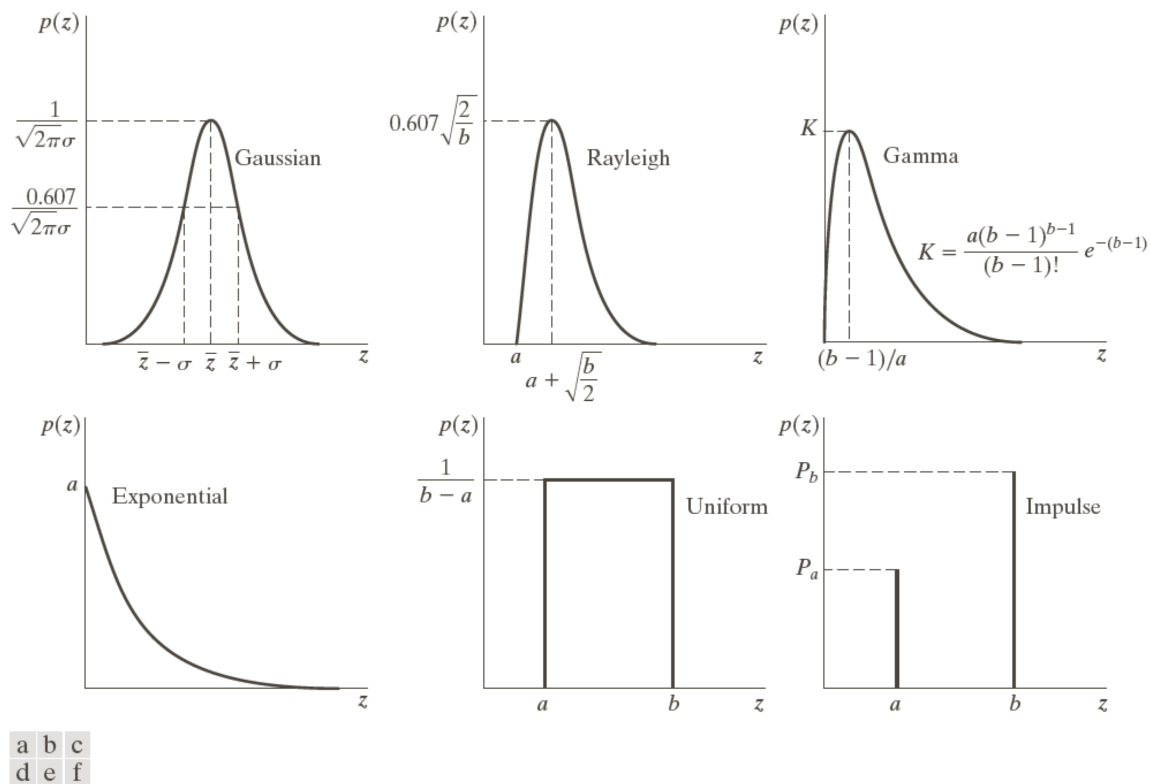


FIGURE 5.2 Some important probability density functions.

Chapter 6

Image Restoration and Reconstruction

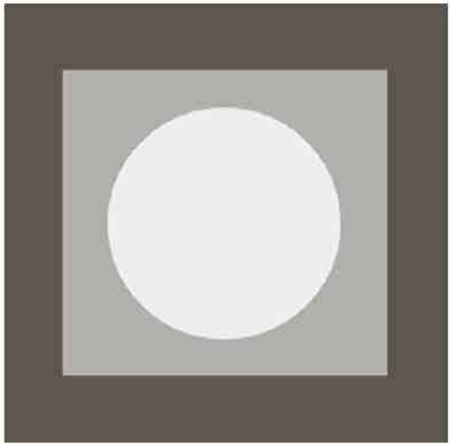


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

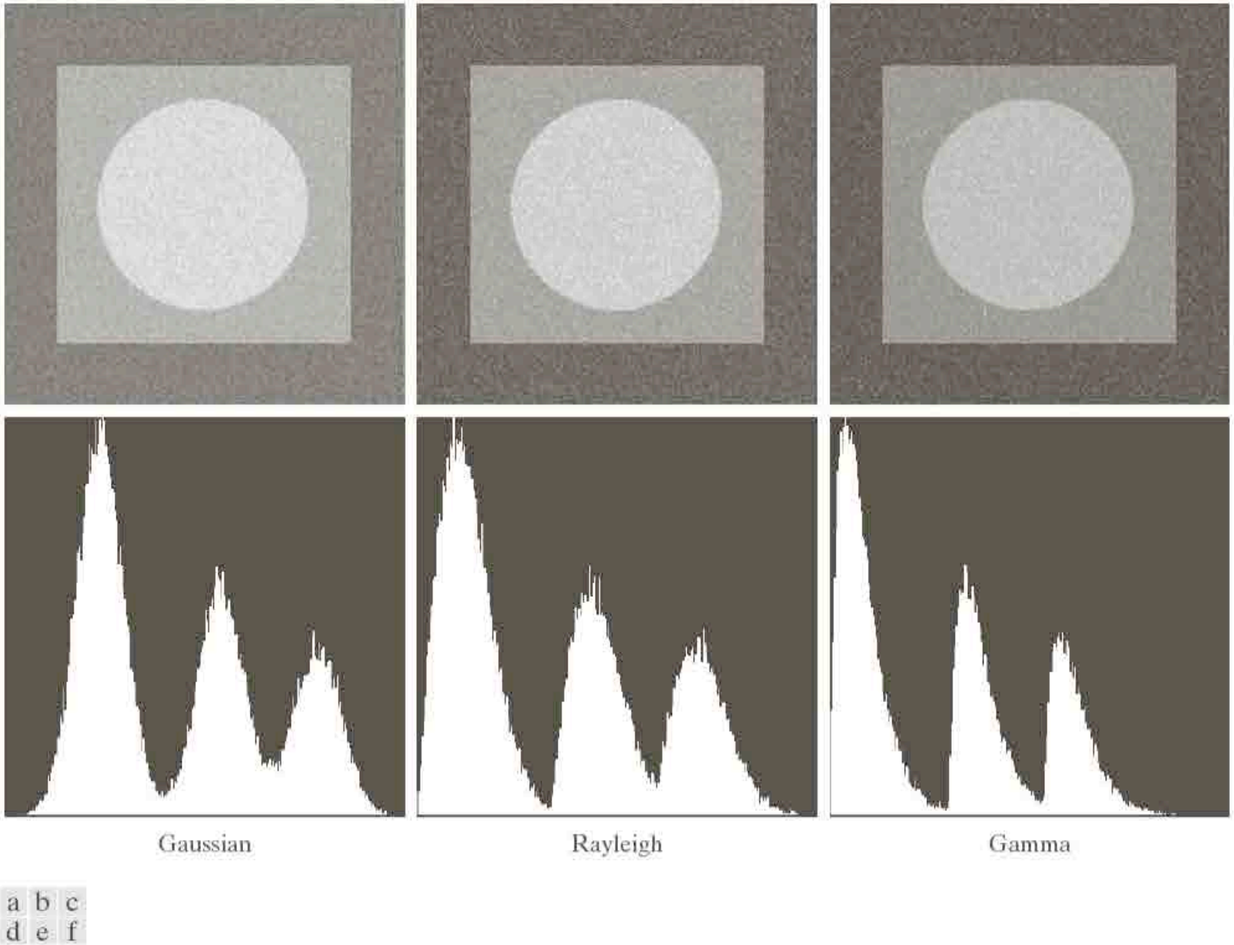


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Chapter 6
Image Restoration and Reconstruction

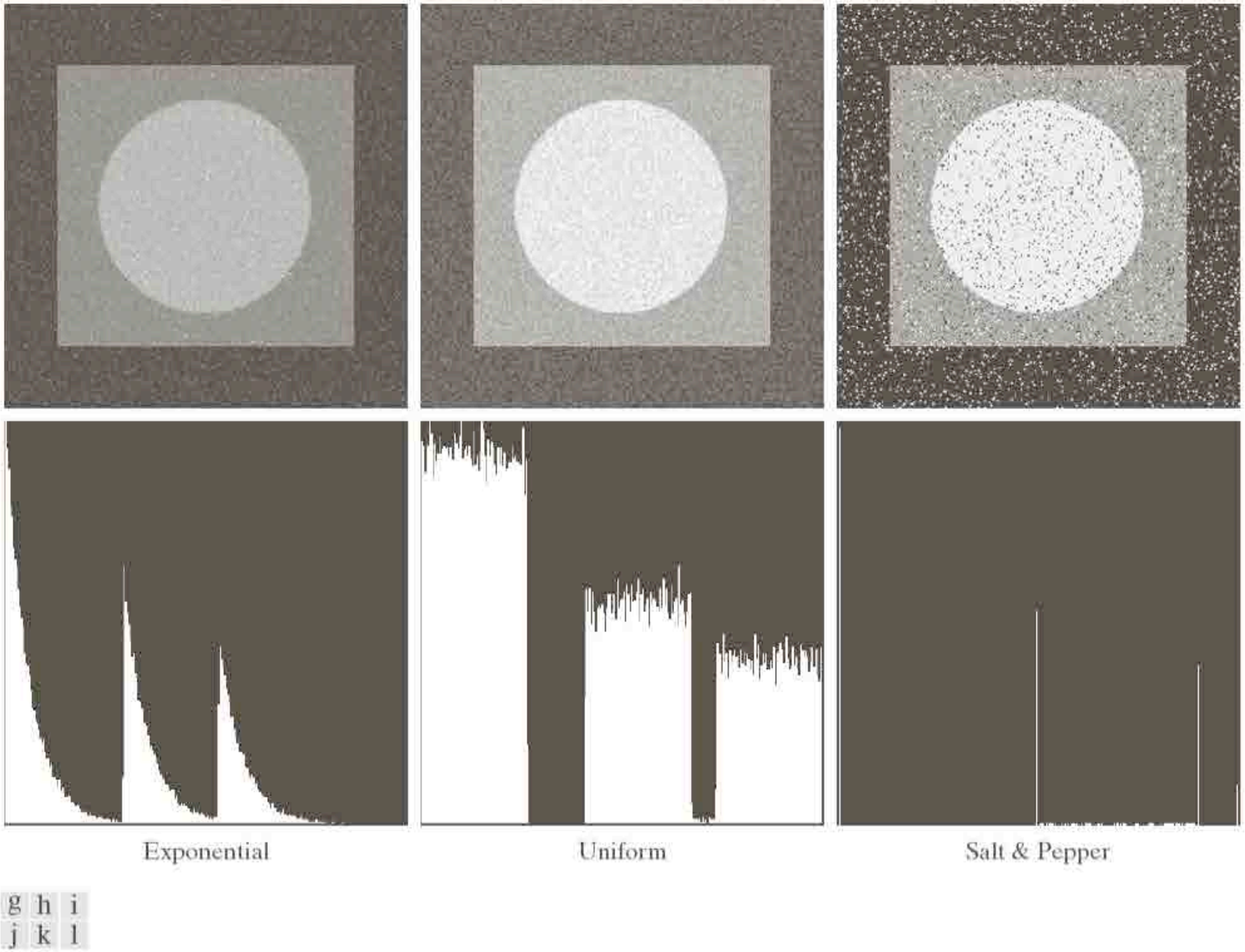
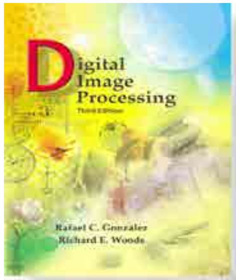
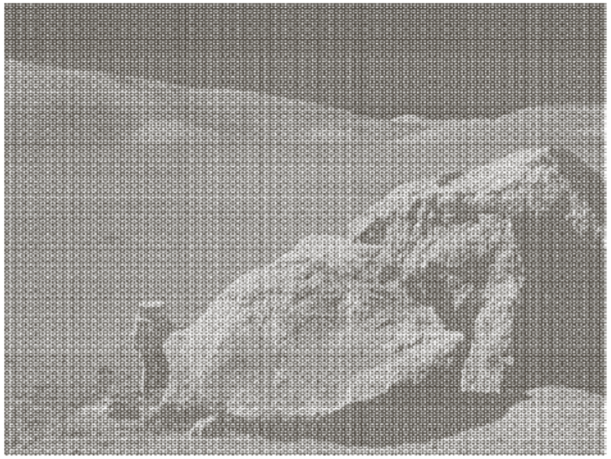


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.



2.3 Periodic Noise

Typically comes from electrical and electromechanical interference during image acquisition



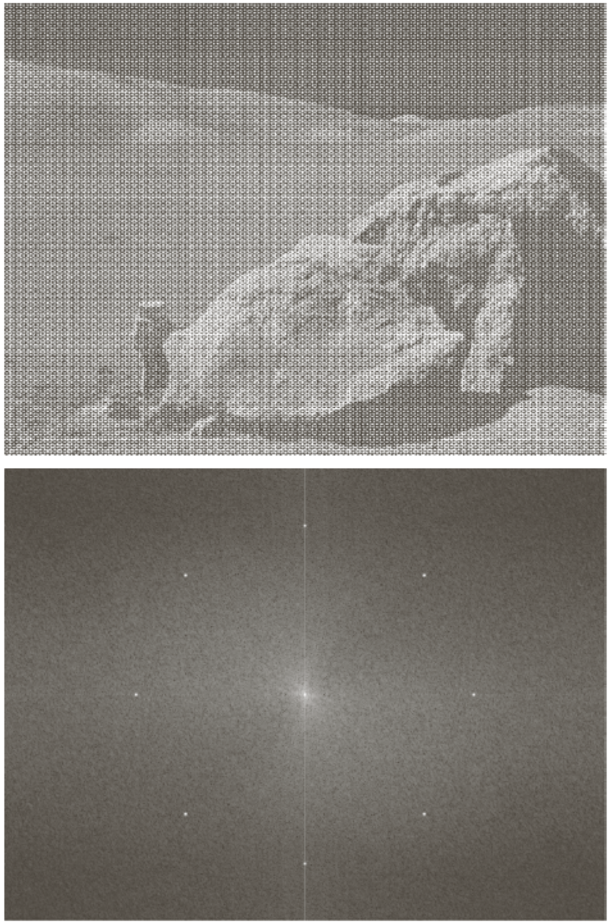
a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise.

2.3 Periodic Noise

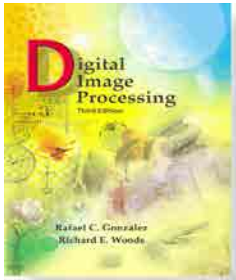
Typically comes from electrical and electromechanical interference during image acquisition

Can be reduced significantly using frequency domain filtering



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)



2.4 Estimation of Noise Parameters

S: sub-image

$p_s(z_i)$: probability estimates of the intensities of pixels in S

L: number of possible intensities in the entire image

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i) \qquad \sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$

a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

3. Restoration in the Presence of Noise Only - Spatial Filtering

When the only degradation is noise, the corrupted image is:

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

When only additive noise present: *spatial filtering*

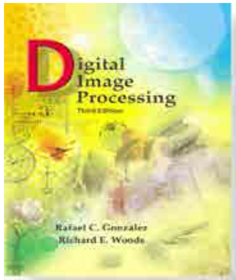
3.1 Mean Filters

<u>Arithmetic mean filter</u>

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s, t)$$

<u>Geometric mean filter</u>

$$\hat{f}(x, y) = \left[\prod_{(s,t)\in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$



3.1 Mean Filters

<u>Harmonic mean filter</u>

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise or Gaussian noise, but fails for pepper noise

<u>Contraharmonic mean filter</u>

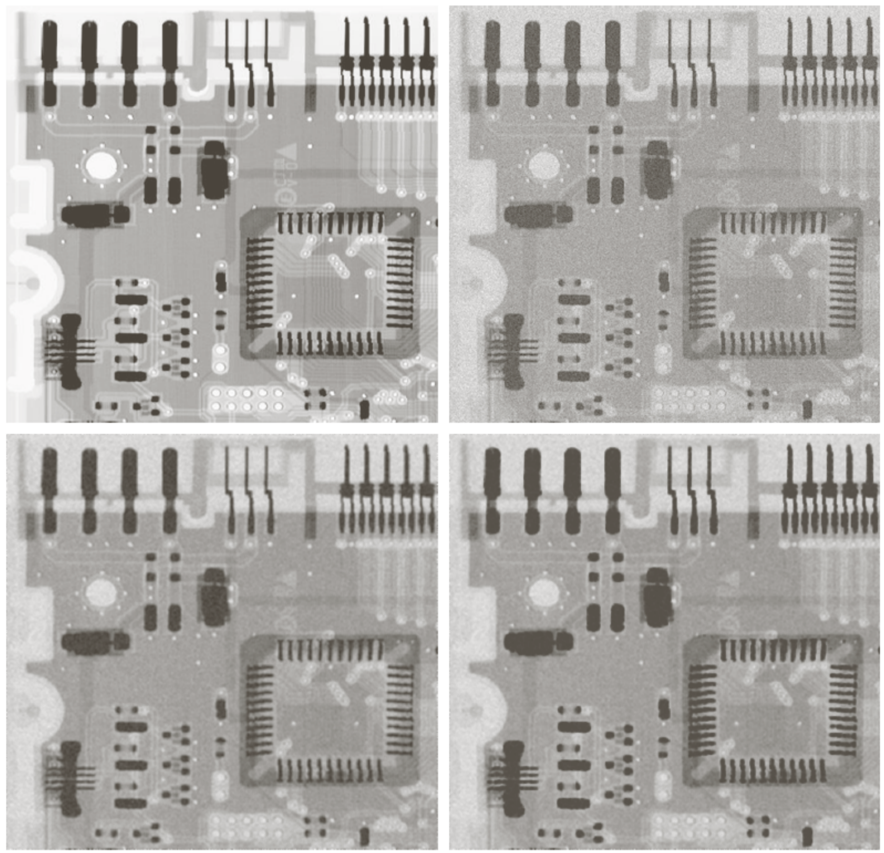
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q = *order* of the filter

Good for salt-and-pepper noise.

Eliminates pepper noise for $Q > 0$ and salt noise for $Q < 0$

NB: cf. arithmetic filter if $Q = 0$, harmonic mean filter if $Q = -1$



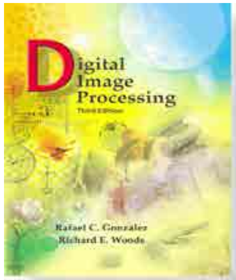
Gaussian noise

3x3 geometric mean filter

3x3 arithmetic mean filter

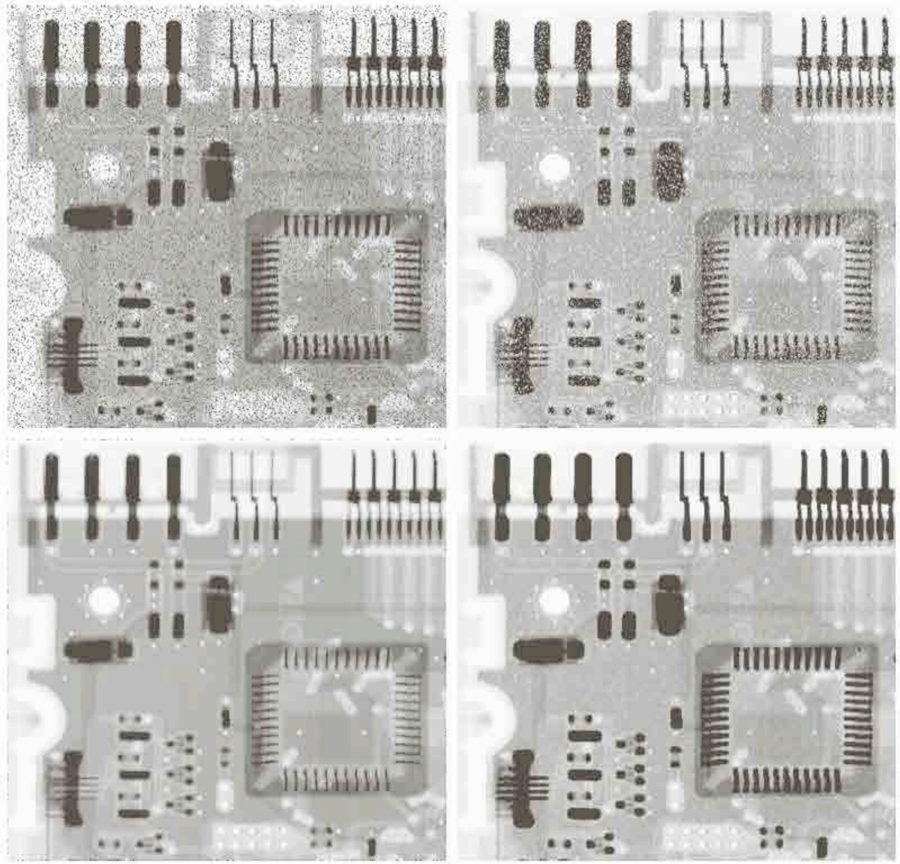
a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Pepper noise, proba = 0.1

Salt noise



a b
c d

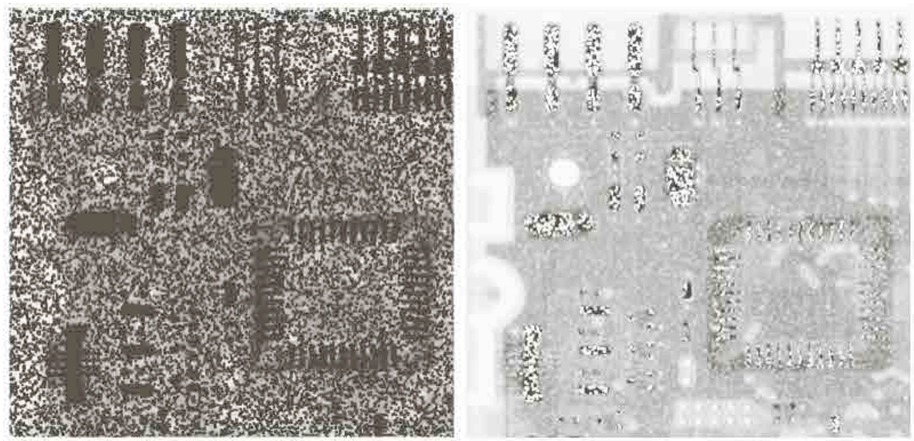
FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

3x3 contraharmonic filter
Q = 1.5

3x3 contraharmonic filter
Q = -1.5

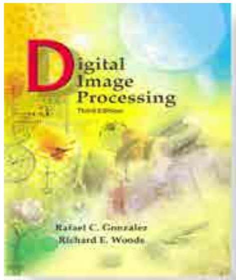
a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.



Filtering pepper noise
with a
3x3 contraharmonic filter
Q = 1.5

Filtering salt noise
with a
3x3 contraharmonic filter
Q = -1.5

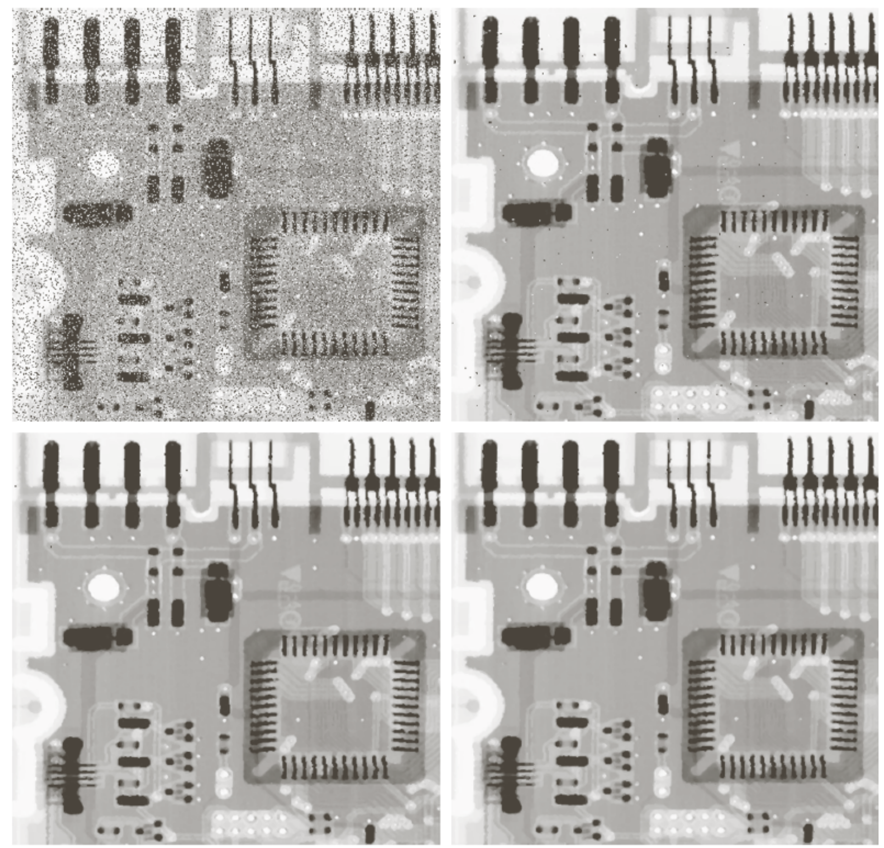


3.2 Order-Statistic Filters

Median filter $\hat{f}(x, y) = \text{median}\{g(s,t)\}_{(s,t)\in S_{xy}}$

Particularly effective with bipolar and unipolar impulse noises

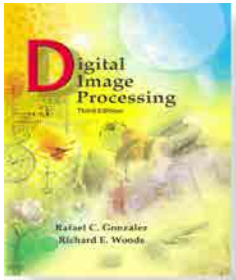
3x3 median filter



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

2d pass



3.2 Order-Statistic Filters

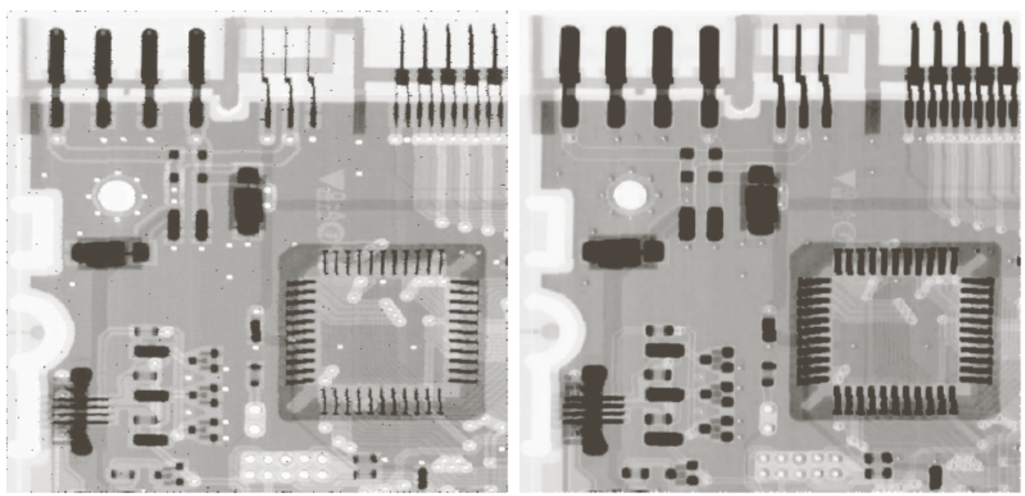
Max filter: $$\hat{f}(x,y) = \max\{g(s,t)\}_{(s,t)\in S_{xy}}$$

Useful for finding the brightest points in an image

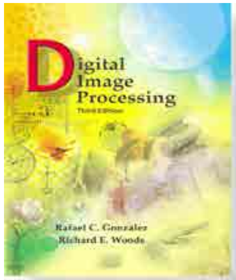
Min filter: $$\hat{f}(x,y) = \min\{g(s,t)\}_{(s,t)\in S_{xy}}$$

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter Min filter



3.2 Order-Statistic Filters

<u>Midpoint filter</u>

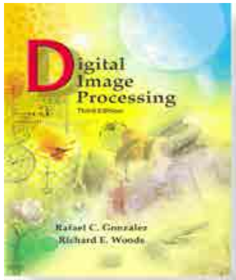
$$\hat{f}(x,y) = \frac{1}{2}\left[\max\{g(s,t)\}_{(s,t)\in S_{xy}} + \min\{g(s,t)\}_{(s,t)\in S_{xy}}\right]$$

NB: combines order statistics and averaging.
Works best for randomly distributed noise such as Gaussian or uniform

<u>Alpha-trimmed mean filter</u>

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

Where g_r represents the image g in which the $d/2$ lowest and $d/2$ highest intensity values in the neighbourhood S_{xy} were deleted
NB: $d = 0$ => arithmetic mean filter, $d = mn-1$ => median filter
For other values of d, useful when multiple types of noise (e.g. combination of salt-and-pepper and Gaussian Noise)

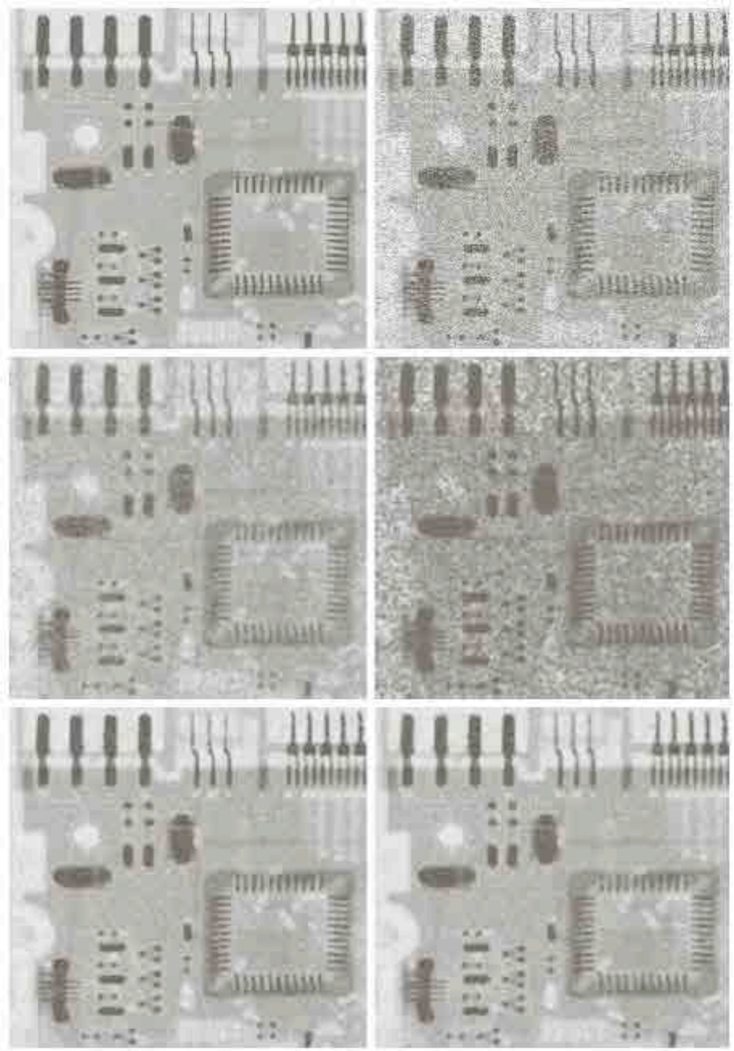


Uniform noise

+ salt-and-pepper noise

Arithmetic mean filter

Geometric mean filter

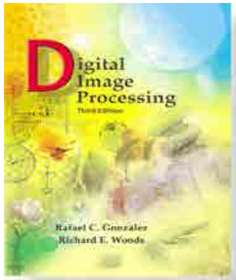


Median filter

Alpha-trimmed mean filter

a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



4 Periodic Noise Reduction by Frequency Domain Filtering

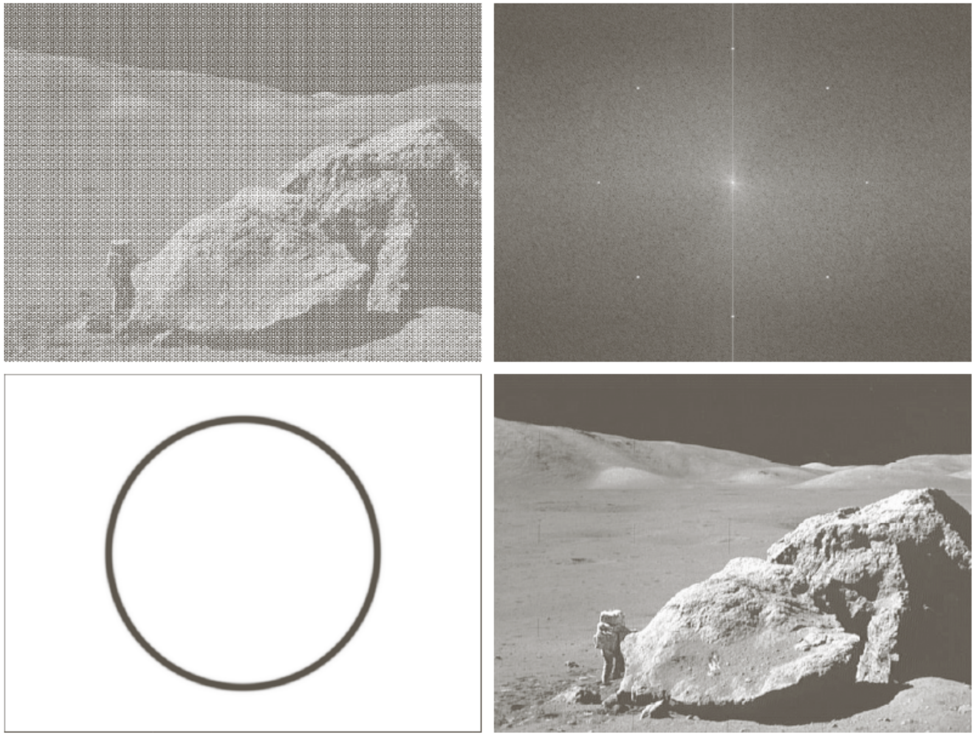
• Periodic noise appears as concentrated bursts of energy in the FT, at locations corresponding to the frequencies of the periodic interference
• Approach: use a selective filter to isolate the noise

3.1 Bandreject Filters
3.2 Bandpass Filters
3.3 Notch Filters : reject (or pass) frequencies in predefined neighbourhoods about a center frequency

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Chapter 6

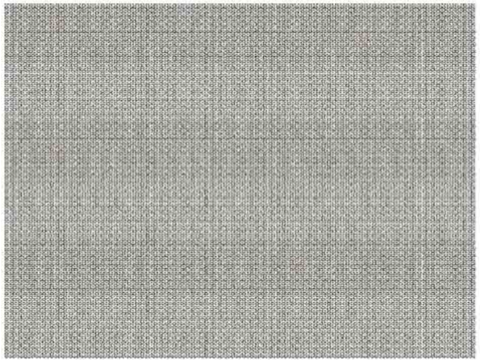
Image Restoration and Reconstruction



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Notch (reject) filters

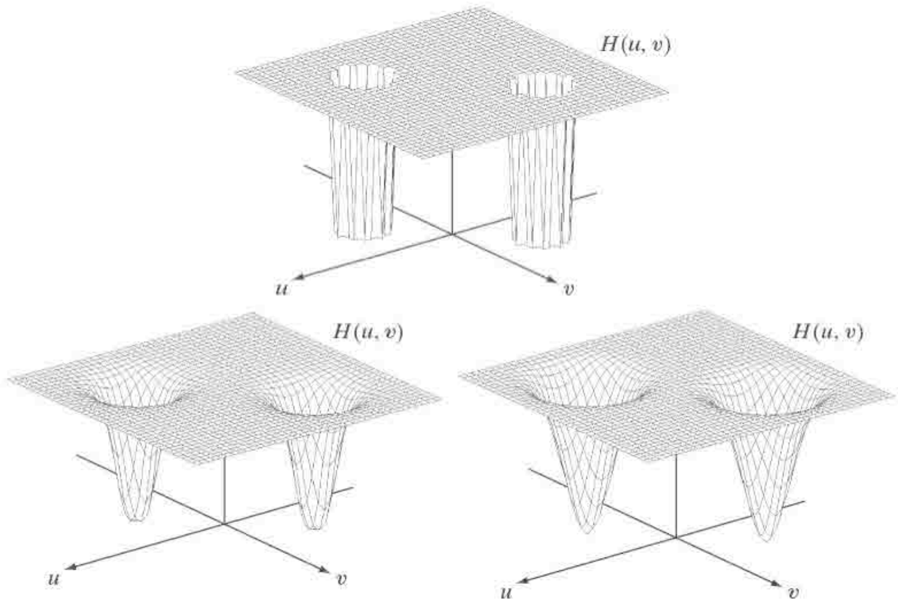
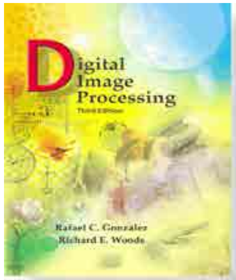


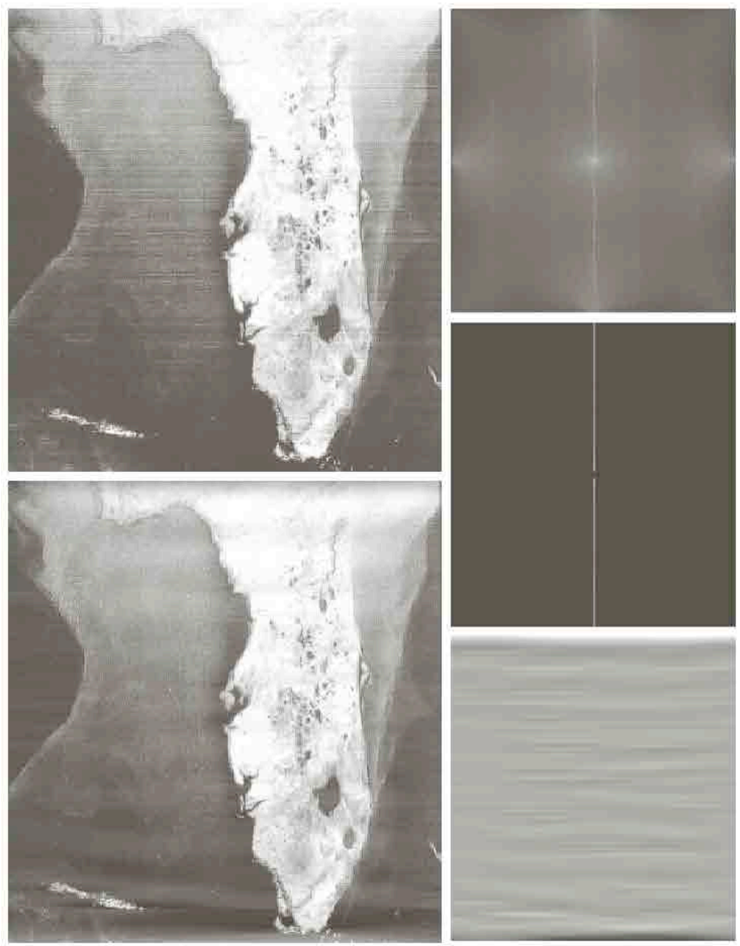
FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



Chapter 6
Image Restoration and Reconstruction

spectrum

Degraded image



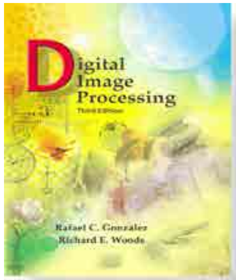
a b c e d

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Notch pass filter

Filtered image

Spatial noise pattern



5 Estimating the Degradation Function

3 main ways to estimate the degradation function for use in an image restoration:

1. Observation
2. Experimentation
3. Mathematical modeling

5.1 Estimation by Image Observation

The degradation is assumed to be *linear* and *position-invariant*

• Look at a small rectangular section of the image containing sample structures, and in which the signal content is strong (e.g. high contrast): subimage $g_s(x,y)$

• Process this subimage to arrive at a result as good as possible: $\hat{f}_s(x, y)$

Assuming the effect of noise is negligible in this area: $H_s(u, v) = \dfrac{G_s(u, v)}{\hat{F}_s(u, v)}$

=> deduce the complete degradation function $H(u,v)$ (position invariance)

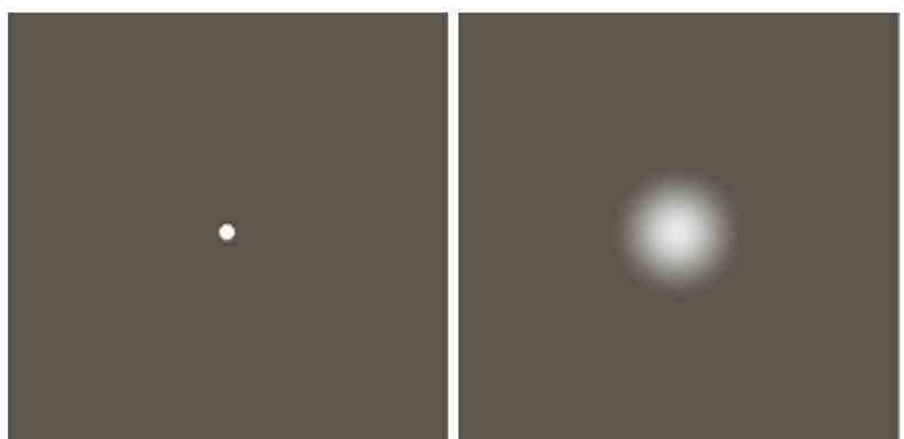
5.2 Estimation by Experimentation

If an equipment similar to the one used to acquire the degraded image is available:

- Find system settings reproducing the most similar degradation as possible
- Obtain an impulse response of the degradation by imaging an impulse (dot of light)

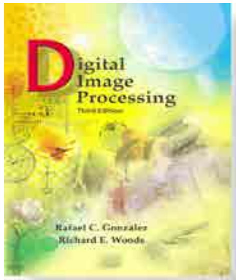
FT of an impulse = constant => $$H(u,v) = \frac{G(u,v)}{A}$$

A = constant describing the strength of the impulse



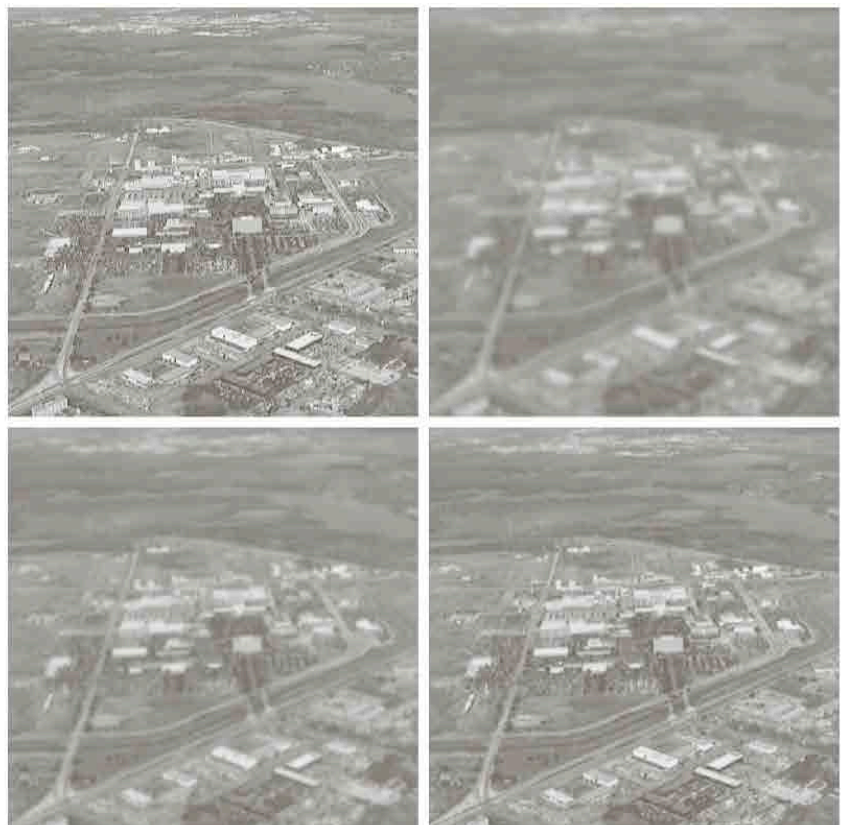
a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



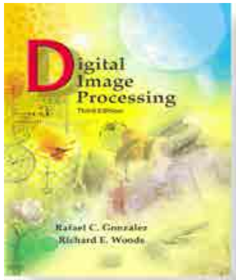
5.3 Estimation by Modeling

Example 1: degradation model proposed by Hufnagel and Stanley [1964] based on the physical characteristics of atmospheric turbulence: $H(u,v) = e^{-k(u^2+v^2)^{5/6}}$



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



Example 2: derive a mathematical model starting form basic principles
Illustration: image blurring by uniform linear motion between the image and the sensor during image acquisition
If T is the duration of exposure the blurred image can be expressed as:

$$g(x,y) = \int_0^T f\left[x - x_0(t), y - y_0(t)\right] dt$$

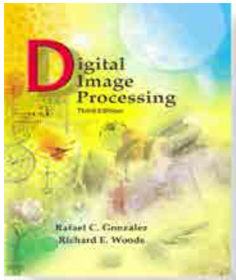
FT[g(x,y)] => $$G(u,v) = F(u,v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt$$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt \qquad \Rightarrow \qquad G(u,v) = H(u,v)F(u,v)$$

E.g. if uniform linear motion in the *x*-direction only, at a rate $x_0(t) = at/T$

$$H(u,v) = \frac{T}{\pi ua} \sin(\pi ua) e^{-j\pi ua}$$

NB: $H = 0$ for $u = n/a$

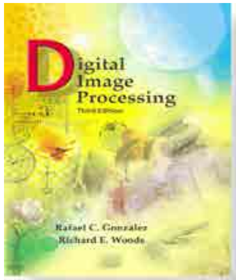


If motion in y as well:

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right] e^{-j\pi(ua+vb)}$$

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.



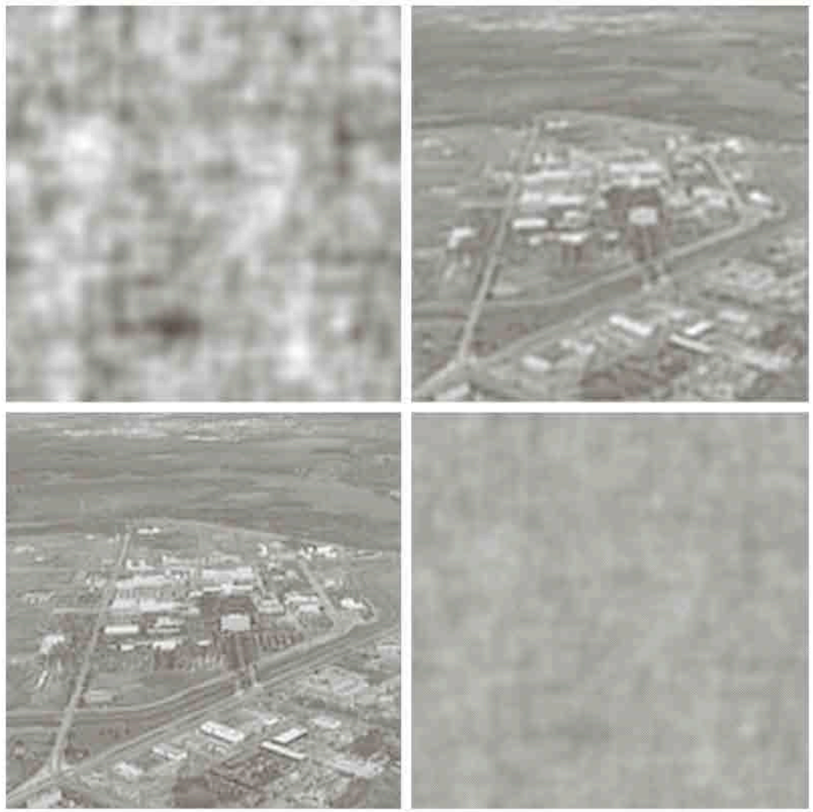
6 Inverse Filtering

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \qquad \text{(array operation)}$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \qquad \Rightarrow \; \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

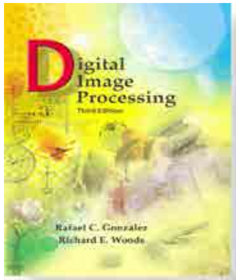
⇒ Even if we know *H(u,v)*, we cannot recover the “undegraded” image exactly because *N(u,v)* is not known
⇒ If *H* has zero or very small values, the ration *N/H* could dominate the estimate

One approach to get around this is to limit the filter frequencies to values near the origin



a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



7. Minimum Mean Square Error (Wiener) Filtering

Objective: find an estimate $\hat{f}$ of the uncorrupted image such that the mean square error between them is minimized: $e^2 = E\{(f - \hat{f})^2\}$

Assumptions:

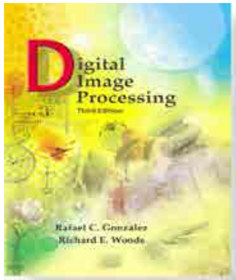
- Noise and image are uncorrelated
- One or the other has zero mean
- The intensity levels in the estimate are a linear function of the levels in the degraded image

The minimum of the error function e is given by:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

$S_\eta(u,v) = |N(u,v)|^2 =$ Power spectrum of the noise (autocorrelation of noise)

$S_f(u,v) = |F(u,v)|^2 =$ Power spectrum of the undegraded image



7. Minimum Mean Square Error (Wiener) Filtering

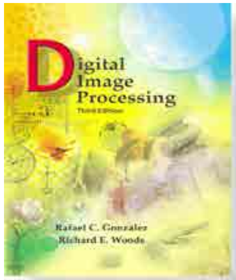
When the two spectrums are not known or cannot be estimated, approximate to:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Where K is a specified constant

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



Chapter 6

Image Restoration and Reconstruction

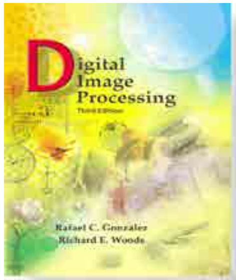
Image + motion blur + add. noise | Result of Inverse filtering | Result of Wiener filtering

Reduced noise variance

Reduced noise variance

a b c
d e f
g h i

FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.



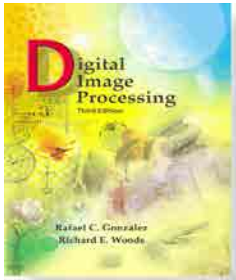
8. Geometric Mean Filter

Generalization of the Wiener filter:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

α and β being positive real constants

α = 1 => inverse filter
α = 0 => *parametric Wiener filter* (standard Wiener filter when β = 1)
α = 1/2 => actual geometric mean
α = 1/2 and β = 1 => *spectrum equalization filter*



9. Constrained Least Squares Filtering

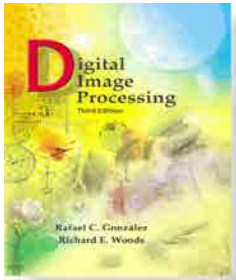
In vector-matrix form: $$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\eta}$$

$\boldsymbol{g}, \boldsymbol{f}, \boldsymbol{\eta}$ vectors of dimension MNx1
$\boldsymbol{H}$ matrix of dimension MNxMN => very large !

Issue: Sensitivity of H to noise
⇒ Optimality of restoration based on a measure of smoothness: e.g. Laplacian
⇒ Find the minimum of a criterion function C:

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\nabla^2 f(x,y)\right]^2$$

subject to the constraint: $||\boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}}||^2 = ||\boldsymbol{\eta}||^2$ (Euclidean vector norm)



9. Constrained Least Squares Filtering

Frequency domain solution:

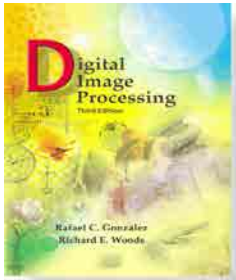
$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

With:

- γ = parameter to adjust so that the constraint is satisfied
- $P(u,v)$ = Fourier Transform of the function:

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

NB: $\gamma = 0$ => Inverse Filtering



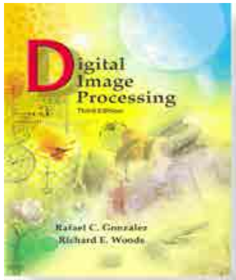
9. Constrained Least Squares Filtering

Results adjusting γ interactively

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

Wiener Filtering

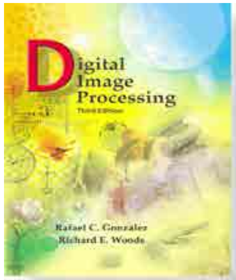


9. Constrained Least Squares Filtering

Adjusting γ so that the constraint is satisfied
(an algorithm)

Goal: find γ so that: $\|\boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}}\|^2 = \|\boldsymbol{\eta}\|^2 \pm a$ (1) (a = accuracy factor)

1. Specify an initial value of γ
2. Compute the corresponding residual $\|\boldsymbol{r}\|^2 = \|\boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}}\|^2$
3. Stop if Eq. (1) is satisfied. Otherwise:
   - if $\|\boldsymbol{r}\|^2 < \|\boldsymbol{\eta}\|^2 - a$, increase γ ,
   - if $\|\boldsymbol{r}\|^2 > \|\boldsymbol{\eta}\|^2 + a$, decrease γ ,
   - Then return to step 2.



9. Constrained Least Squares Filtering

Adjusting γ so that the constraint is satisfied

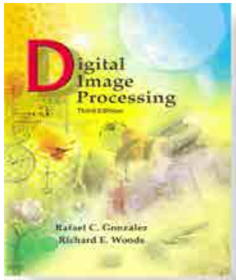
$$||\boldsymbol{r}||^2 = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} r^2(x,y)$$

$$||\boldsymbol{\eta}||^2 = MN\left[\sigma_n^2 + m_n^2\right]$$

where:

$$m_n = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \eta(x,y) \qquad \text{(average)}$$

$$\sigma_n^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[\eta(x,y) - m_n\right]^2 \qquad \text{(variance)}$$



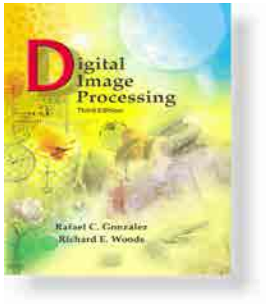
9. Constrained Least Squares Filtering

Adjusting γ so that the constraint is satisfied

Restoration of:

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.



Chapter 6
Image Restoration and Reconstruction

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