$$\therefore \qquad p = \frac{30}{4} = 15 \text{ N/mm}^2 = 15 \text{ MPa}$$

Now we shall provide a pressure of 7.5 MPa *i.e.* (Lesser of the two values) obtained by using the tensile stress as circumferential stress and longitudinal stress.

31.6. Design of Thin Cylindrical Shells

Designing of thin cylindrical shell involves calculating the thickness (t) of a cylindrical shell for the given length (l), diameter (d), intensity of maximum internal pressure (p) and circumferential stress (σ_c). The required thickness of the shell is calculated from the relation.

$$t = \frac{pd}{2\sigma_c} \qquad \text{... (See Article 31.4)}$$

If the thickness so obtained, is not a round figure, then next higher value is provided.

Note: The thickness obtained from the longitudinal stress will be half of the thickness obtained from circumferential stress. Thus, it should not be accepted.

Example 31.4. *A thin cylindrical shell of 400 mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of the shell, if the allowable circumferential stress is 50 MPa.*

Solution. Given: Diameter of shell (d) = 400 mm ; Internal pressure (p) = 2.4 MPa = 2.4 N/mm^2 and circumferential stress (σ_c) = 50 MPa = 50 N/mm^2.

We know that thickness of the shell,

$$t = \frac{pd}{2\sigma_c} = \frac{2.4 \times 400}{2 \times 50} = \mathbf{9.6 \text{ mm} \quad say \quad 10 \text{ mm}} \qquad \textbf{Ans.}$$

Example 31.5. *A cylindrical shell of 500 mm diameter is required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa and efficiency of the joints is 65%. Take factor of safety as 5.*

Solution. Given: Diameter of shell (d) = 500 mm ; Internal pressure (p) = 4 MPa = 4 N/mm^2; Tensile strength = 400 MPa = 400 N/mm^2 ; Efficiency (η) = 65% = 0.65 and factor of safety = 5.

We know that allowable tensile stress (*i.e.*, circumferential stress),

$$\sigma_c = \frac{\text{Tensile strength}}{\text{Factor of safety}} = \frac{400}{5} = 80 \text{ N/mm}^2$$

and minimum thickness of shell,

$$t = \frac{pd}{2\sigma_c \eta} = \frac{4 \times 500}{2 \times 80 \times 0.65} = \mathbf{19.2 \text{ mm} \quad say \quad 20 \text{ mm}} \qquad \textbf{Ans.}$$

31.7. Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

We have already discussed in the chapter on Elastic Constants that lateral strain is always accompanied by a linear strain. It is thus obvious that in a thin cylindrical shell subjected to an internal pressure, its walls will also be subjected to lateral strain. The effect of the lateral strains is to cause some change in the dimensions (*i.e.*, length and diameter) of the shell. Now consider a thin cylindrical shell subjected to an internal pressure.

Let l = Length of the shell,

d = Diameter of the shell,

t = Thickness of the shell and

p = Intensity of the internal pressure.

We know that the circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

and longitudinal stress,

$$\sigma_l = \frac{pd}{4t}$$

Now let δd = Change in diameter of the shell,

δl = Change in the length of the shell and

$\frac{1}{m}$ = Poisson's ratio.

Now changes in diameter and length may be found out from the above equations, as usual (*i.e.*, by multiplying the strain and the corresponding linear dimension).

$$\therefore \quad \delta d = \varepsilon_1 \cdot d = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) \times d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

and

$$\delta l = \varepsilon_2 \cdot l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) \times l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

EXAMPLE 31.6. *A cylindrical thin drum 800 mm in diameter and 4 m long is made of 10 mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa, determine its changes in diameter and length. Take E as 200 GPa and Poisson's ratio as 0.25.*

SOLUTION. Given: Diameter of drum (d) = 800 mm ; Length of drum (l) = 4 m = 4×10^3 mm ; Thickness of plates (t) = 10 mm ; Internal pressure (p) = 2.5 MPa = 2.5 N/mm^2 ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and poisson's ratio $\left(\frac{1}{m}\right)$ = 0.25.

Change in diameter

We know that change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right) = \frac{2.5 \times (800)^2}{2 \times 10 \times (200 \times 10^3)}\left(1 - \frac{0.25}{2}\right) \text{mm}$$

$$= \mathbf{0.35\ mm} \qquad \textbf{Ans.}$$

Change in length

We also know that change in length,

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{2.5 \times 800 \times (4 \times 10^3)}{2 \times 10 \times (200 \times 10^3)}\left(\frac{1}{2} - 0.25\right) \text{mm}$$

$$= \mathbf{0.5\ mm} \qquad \textbf{Ans.}$$

31.8. Change in Volume of a Thin Cylindrical Shell due to an Internal Pressure

We have already discussed in the last article, that there is always an increase in the length and diameter of a thin cylindrical shell due to an internal pressure. A little consideration will show that increase in the length and diameter of the shell will also increase its volume. Now consider a thin cylindrical shell subjected to an internal pressure.

Let l = Original length,

d = Original diameter,

δl = Change in length due to pressure and

δd = Change in diameter due to pressure.

We know that original volume,

$$V = \frac{\pi}{4} \times d^2 \times l = \left[\frac{\pi}{4}(d + \delta d)^2 \times (l \times \delta l)\right] - \frac{\pi}{4} \times d^2 \times l$$

$$= \frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d) \qquad \text{...(Neglecting small quantities)}$$

$$\therefore \quad \frac{\delta V}{V} = \frac{\frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d)}{\frac{\pi}{4} \times d^2 \times l} = \frac{\delta l}{l} + \frac{2\delta d}{d} = \varepsilon_l + 2\varepsilon_c$$

or $$\delta V = V(\varepsilon_l + 2\varepsilon_c)$$

where ε_c = Circumferential strain and

ε_l = Longitudinal strain.

EXAMPLE 31.7. *A cylindrical vessel 2 m long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E = 200 GPa and Poisson's ratio = 0.3 for the vessel material.*

SOLUTION. Given: Length of vessel (l) = 2 m = 2×10^3 mm ; Diameter of vessel (d) = 500 mm ; Thickness of plates (t) = 10 mm ; Internal pressure (p) = 3 MPa = 3 N/mm^2 ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and poisson's ratio $\left(\frac{1}{m}\right)$ = 0.3.

We know that circumferential strain,

$$\varepsilon_c = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) = \frac{3 \times 500}{2 \times 10 \times (200 \times 10^3)}\left(1 - \frac{0.3}{2}\right) = 0.32 \times 10^{-3} \qquad \text{...(i)}$$

and logitudinal strain,

$$\varepsilon_l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{3 \times 500}{2 \times 10 \times (200 \times 10^3)}\left(\frac{1}{2} - 0.3\right) = 0.075 \times 10^{-3} \qquad \text{...(ii)}$$

We also know that original volume of the vessel,

$$V = \frac{\pi}{4}(d)^2 \times l = \frac{\pi}{4}(500)^2 \times (2 \times 10^3) = 392.7 \times 10^6 \text{ mm}^3$$

∴ Change in volume,

$$\delta V = V(\varepsilon_c + 2\varepsilon_l) = 392.7 \times 10^6 [0.32 \times 10^{-3} + (2 \times 0.075 \times 10^{-3})] \text{ mm}^3$$

$$= \mathbf{185 \times 10^3 \text{ mm}^3} \qquad \textbf{Ans.}$$

EXERCISE 31.1

1. A cylindrical shell 2 m long and 1 m internal diameter is made up of 20 mm thick plates. Find the circumferential and longitudinal stresses in the shell material, if it is subjected to an internal pressure of 5 MPa. (**Ans.** 125 MPa ; 62.5 MPa)
2. A steam boiler of 1.25 m in diameter is subjected to an internal pressure of 1.6 MPa. If the steam boiler is made up of 20 mm thick plates, calculate the circumferential and longitudinal stresses. Take efficiency of the circumferential and longitudinal joints as 75% and 60% respectively. (**Ans.** 67 MPa ; 42 MPa)
3. A pipe of 100 mm diameter is carrying a fluid under a pressure of 4 MPa. What should be the minimum thickness of the pipe, if maximum circumferential stress in the pipe material is 12.5 MPa. (**Ans.** 16 mm)

4. A cylindrical shell 3 m long has 1 m internal diameter and 15 mm metal thickness. Calculate the circumferential and longitudinal stresses, if the shell is subjected to an internal pressure of 1.5 MPa. Also calculate the changes in dimensions of the shell. Take E = 200 GPa and Poisson's ratio = 0.3. (**Ans.** 50 MPa ; 25 MPa ; δd = 0.21 mm ; δl = 0.15 mm)

5. A cylindrical vessel 1.8 m long 800 mm in diameter is made up of 8 mm thick plates. Find the hoop and longitudinal stresses in the vessel, when it contains fluid under a pressure of 2.5 MPa. Also find the changes in length, diameter and volume of the vessel. Take E = 200 GPa and $1/m$ = 0.3. (**Ans.** 125 MPa ; 62.5 MPa ; 0.42 mm ; 0.23 mm ; 1074 mm^3)

31.9. Thin Spherical Shells

Consider a thin spherical shell subjected to an internal pressure as shown in Fig. 31.4.

Let p = Intensity of internal pressure,

d = Diameter of the shell and

t = Thickness of the shell,

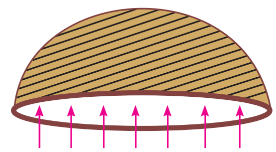

Fig. 31.4. Spherical shell

As a result of this internal pressure, the shell is likely to be torn away along the centre of the sphere. Therefore, total pressure acting along the centre of the sphere,

$$P = \text{Intensity of internal pressure} \times \text{Area}$$

$$= p \times \frac{\pi}{4} \times d^2$$

and stress in the shell material,

$$\sigma = \frac{\text{Total pressure}}{\text{Resisting section}} = \frac{p \times \frac{\pi}{4} \times d^2}{\pi d \times t} = \frac{pd}{4t}$$

Note. If η is the efficiency of the riveted joints of the spherical shell, then stress,

$$\sigma = \frac{pd}{4t\eta}$$

EXAMPLE 31.8. *A spherical gas vessel of 1.2 m diameter is subjected to a pressure of 1.8 MPa. Determine the stress induced in the vessel plate, if its thickness is 5 mm.*

SOLUTION. Given: Diameter of vessel (d) = 1.2 m = 1.2×10^3 mm ; Internal pressure (p) = 1.8 MPa = 1.8 N/mm^2 and thickness of plates (t) = 5 mm.

We know that stress in the vessel plates,

$$\sigma = \frac{pd}{4t} = \frac{1.8 \times (1.2 \times 10^3)}{4 \times 5} = 108 \text{ N/mm}^2 = \mathbf{108 \text{ MPa}} \qquad \textbf{Ans.}$$

EXAMPLE 31.9. *A spherical vessel of 2 m diameter is subjected to an internal pressure of 2 MPa. Find the minimum thickness of the plates required, if the maximum stress is not to exceed 100 MPa. Take efficiency of the joint as 80%.*

SOLUTION. Given: Diameter of vessel (d) = 2 m = 2×10^3 mm ; Internal pressure (p) = 2 MPa = 2 N/mm^2 ; Maximum stress (σ) = 100 MPa = 100 N/mm^2 and efficiency of joint (η) = 80% = 0.8.

Let t = Minimum thickness of the plates in mm.

We know that stress in the plates (σ),

$$100 = \frac{pd}{4t\,\eta} = \frac{2 \times (2 \times 10^3)}{4 \times t \times 0.8} = \frac{1250}{t}$$

$$\therefore \quad t = \frac{1250}{100} = \mathbf{12.5 \text{ mm}} \qquad \textbf{Ans.}$$

31.10. Change in Diameter and Volume of a Thin Spherical Shell due to an Internal Pressure

Consider a thin spherical shell subjected to an internal pressure.

Let d = Diameter of the shell,

p = Intensity of internal pressure and

t = Thickness of the shell.

We have already discussed in the last article that the stress in a spherical shell,

$$\sigma = \frac{pd}{4t}$$

and strain in any one direction,

$$\varepsilon = \frac{\sigma}{E} - \frac{\sigma}{mE} \qquad ...(\because \sigma_1 = \sigma_2 = \sigma)$$

$$= \frac{pd}{4tE} - \frac{pd}{4tEm} = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$$

∴ Change in diameter,

$$\delta d = \varepsilon \cdot d = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) \times d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

We also know that original volume of the sphere,

$$V = \frac{\pi}{6} \times (d)^3$$

and final volume due to pressure,

$$V + \delta V = \frac{\pi}{6} \times (d + \delta d)^3$$

where $(d + \delta d)$ = Final diameter of the shell.

∴ Volumetric strain,

$$\frac{\delta V}{V} = \frac{(V + \delta V) - V}{V} = \frac{\frac{\pi}{6}(d + \delta d)^3 - \frac{\pi}{6} \times d^3}{\frac{\pi}{6} \times d^3}$$

$$= \frac{d^3 + (3d^2 \cdot \delta d) - d^3}{d^3}$$

...(Ignoring second and higher power of δd)

$$= \frac{3 \cdot \delta d}{d} = 3\varepsilon$$

and

$$\delta V = V \cdot 3\varepsilon = \frac{\pi}{6}(d)^3 \times 3 \times \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

EXAMPLE 31.10. *A spherical shell of 2 m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and 1/m = 0.3.*

SOLUTION. Given: Diameter of shell (d) = 2 m = 2×10^3 mm ; Thickness of plates (t) = 10 mm ; Internal pressure (p) = 1.6 MPa = 1.6 N/mm^2 ; Modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2 and Poisson's ratio ($1/m$) = 0.3.

Change in diameter

We know that change in diameter,

$$\delta d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right) = \frac{1.6 \times (2 \times 10^3)^2}{4 \times 10 \times (200 \times 10^3)}(1 - 0.3)$$

$$= \mathbf{0.56\ mm} \qquad \textbf{Ans.}$$

Change in volume

We also know that change in volume,

$$\delta V = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right) = \frac{\pi \times 1.6 \times (2 \times 10^3)^4}{8 \times 10 \times (200 \times 10^3)}(1 - 0.3) \text{ mm}^3$$

$$= \mathbf{3.52 \times 10^6 \text{ mm}^3} \quad \textbf{Ans.}$$

31.11. Riveted Cylindrical Shells

Sometimes, boilers of the desired capacity are made of cylindrical shape by joining different plates usually by rivets. This is generally done : (*i*) by bending the plates to the required diameter and then joining them by a butt joint and (*ii*) by joining individually fabricated shells by a lap joint as shown in Fig. 31.5 (*a*) and (*b*). A little consideration will show that in this case, the plate is weakened by the rivet hole.

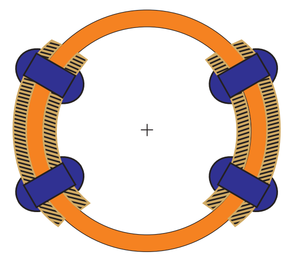

Fig. 31.5 (*a*) Joining by Butt Joint

The circumferential stress in a riveted cylindrical shell,

$$\delta_c = \frac{pd}{2t\eta}$$

Similarly, longitudinal stress,

$$\delta_l = \frac{pd}{4t\eta}$$

where η is the efficiency of the riveted joint.

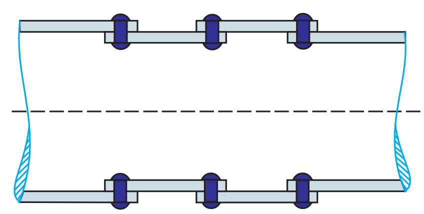

Fig. 31.5 (*b*) Joining by Lap Joint

NOTES: 1. If the efficiency of the joint is different *i.e.*, the joint has different longitudinal efficiency and circumferential efficiency, then the respective values should be used in the above relation.

2. For designing the shell *i.e.*, determining the thickness of shell, the efficiency of the joint should also be considered.

EXAMPLE 31.11. *A boiler shell of 2 m diameter is made up of mild steel plates of 20 mm thick. The efficiency of the longitudinal and circumferential joints is 70% and 60% respectively. Determine the safe pressure in the boiler, if the permissible tensile stress in the plate section through the rivets is 100 MPa. Also determine the circumferential stress in the plate and longitudinal stress through the rivets.*

SOLUTION. Given: Diameter of boiler (d) = 2 m = 2×10^3 mm ; Thickness (t) = 20 mm ; Longitudinal efficiency (η_l) = 70% = 0.7 ; Circumferential efficiency (η_c) = 60% = 0.6 and permissible stress (σ) = 100 MPa = 100 N/mm^2.

Safe pressure in boiler

Let p = Safe pressure in boiler in N/mm^2

We know that permissible stress in boiler (σ),

$$100 = \frac{pd}{2t\eta_l} = \frac{p \times (2 \times 10^3)}{2 \times 20 \times 0.7} = \frac{500p}{7}$$

$$p = \frac{100 \times 7}{500} = 1.4 \text{ N/mm}^2 = \mathbf{1.4 \text{ MPa}} \quad \textbf{Ans.}$$