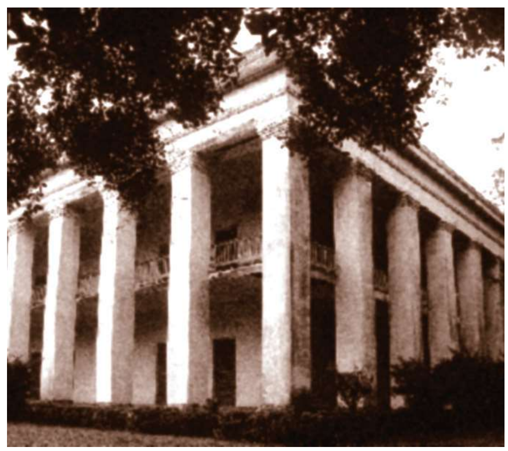
Chapter 34

Columns and Struts

Contents



34.1. Introduction

A structural member, subjected to an axial compressive force, is called a strut. As per definition, a strut may be horizontal, inclined or even vertical. But a vertical strut, used in buildings or frames, is called a *column*.

34.2. Failure of a Column or Strut

It has been observed, that when a column or a strut is subjected to some compressive force, then the compressive stress induced,

$$\sigma = \frac{P}{A}$$

where P = Compressive force and

A = Cross-sectional area of the column.

A little consideration will show, that if the force or load is gradually increased the column will reach a stage, when it will be subjected to the ultimate crushing stress. Beyond this stage, the column will fail by crushing. The load corresponding to the crushing stress, is called *crushing load*.

It has also been experienced that sometimes, a compression member does not fail entirely by crushing, but also by bending *i.e.*, buckling. This happens in the case of long columns. It has also been observed that all the short columns fail due to their crushing. But, if a long column is subjected to a compressive load, it is subjected to a compressive stress. If the load is gradually increased, the column will reach a stage, when it will start buckling. The load, at which the column just buckles is called *buckling load, criticial load or crippling load* and the column is said to have developed an elastic instability. A little consideration will show that for a long column, the value of buckling load will be less than the crushing load. Moreover, the value of buckling load is low for long columns and relatively high for short columns.

34.3. Euler's Column Theory

The first rational attempt, to study the stability of *long columns, was made by Mr. Euler. He derived an equation, for the buckling load of long columns based on the bending stress. While deriving this equation, the effect of direct stress is neglected. This may be justified with the statement that the direct stress induced in a long column is negligible as compared to the bending stress. It may be noted that the Euler's formula cannot be used in the case of short columns, because the direct stress is considerable and hence cannot be neglected.

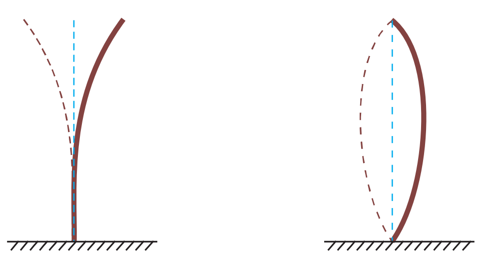
34.4. Assumptions in the Euler's Column Theory

The following simplifying assumptions are made in the Euler's column theory:

1. Initially the column is perfectly straight and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.

34.5. Sign Conventions

Though there are different signs used for the bending of columns in different books, yet we shall follow the following sign conventions which are commonly used and internationally recognised.



(*a*) Positive (*b*) Negative

Fig. 34.1

* As a matter of fact, mere length is not the only criterion for a column to be called long or short. But it has an important relation with the lateral dimensions of the column.

1. A moment, which tends to bend the column with *convexity* towards its initial central line as shown in Fig. 34.1 (*a*) is taken as *positive*.
2. A moment, which tends to bend the column with its *concavity* towards its initial central line as shown in Fig. 34.1 (*b*) is taken as *negative*.

34.6. Types of End Conditions of Columns

In actual practice, there are a number of end conditions, for columns. But, we shall study the Euler's column theory on the following four types of end conditions, which are important from the subject point of view:

1. Both ends hinged,
2. Both ends fixed,
3. One end is fixed and the other hinged, and
4. One end is fixed and the other free.

Now we shall discuss the value of critical load for all the above mentioned type of and conditions of columns one by one.

34.7. Columns with Both Ends Hinged

Consider a column AB of length l hinged at both of its ends A and B and carrying a critical load at B. As a result of loading, let the column deflect into a curved form AX_1B as shown in Fig. 34.2.

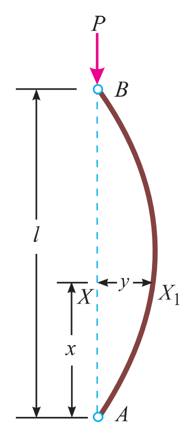


Fig. 34.2

Now consider any section X, at a distance x from A.

Let P = Critical load on the column,

y = Deflection of the column at X.

∴ Moment due to the critical load P,

$$M = -P \cdot y$$

∴ $$EI\frac{d^2y}{dx^2} = -P \cdot y$$ ... (Minus sign due to concavity towards initial centre line)

∴ $$EI\frac{d^2y}{dx^2} + P \cdot y = 0$$

or $$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

The general solution of the above differential equation is

$$y = A \cdot \cos\left(x\sqrt{\frac{P}{EI}}\right) + B\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

where A and B are the constants of integration. We know that when $x = 0$, $y = 0$. Therefore $A = 0$. Similarly when $x = l$, then $y = 0$. Therefore

$$0 = B\sin\left(l\sqrt{\frac{P}{EI}}\right)$$

A little consideration will show that either B is equal to zero or $\sin\left(l\sqrt{\frac{P}{EI}}\right)$ is equal to zero. Now if we consider B to be equal to zero, then it indicates that the column has not bent at all. But if

$$\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$\therefore \quad l\sqrt{\frac{P}{EI}} = 0 = \pi = 2\pi = 3\pi = ...$$

Now taking the least significant value,

$$l\sqrt{\frac{P}{EI}} = \pi$$

or

$$P = \frac{\pi^2 EI}{l^2}$$

34.8. Columns with One End Fixed and the Other Free

Consider a column AB of length l fixed at A and free at B and carrying a critical load at B. As a result of loading, let the beam deflect into a curved form AX_1B_1 such that the free end B deflects through a and occupies a new position B_1 as shown in Fig. 34.3.

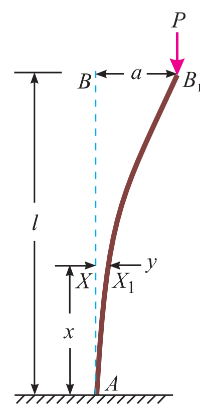


Fig. 34.3

Now consider any section X at a distance x from A.

Let $\quad P$ = Critical load on the column and

$\quad y$ = Deflection of the column at X.

∴ Moment due to the critical load P,

$$M = +P(a - y)$$
$$= P \cdot a - P \cdot y \qquad \text{... (Plus sign due to convexity towards initial centre line)}$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = P \cdot a - P \cdot y$$

or

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{P \cdot a}{EI}$$

The general solution of the above differential equation is,

$$y = A\cos\left(x\sqrt{\frac{P}{EI}}\right) + B\sin\left(x\sqrt{\frac{P}{EI}}\right) + a \qquad ...(i)$$

where A and B are the constants of integration. We know that when $x = 0$, then $y = 0$, therefore $A = -a$. Now differentiating the above equation,

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + B\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right)$$

We also know that when $x = 0$, then $\frac{dy}{dx} = 0$. Therefore

$$0 = B\sqrt{\frac{P}{EI}}$$

A little consideration will show that either B is equal to zero of $\sqrt{\frac{P}{EI}}$ is equal to zero. Since the load P is not equal to zero, it is thus opbvious that B is equal to zero. Now substituting the values $A = -a$ and $B = 0$ is equation (*i*),

$$y = -a\cos\left(x\sqrt{\frac{P}{EI}}\right) + a = a\left[1-\cos\left(x\sqrt{\frac{P}{EI}}\right)\right]$$

We also know that when $x = l$, then $y = a$. Therefore

$$a = a\left[1-\cos\left(l\sqrt{\frac{P}{EI}}\right)\right]$$

$$\therefore \quad \cos\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

or
$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2} = \frac{3\pi}{2} = \frac{5\pi}{2}$$

Now taking the least significant value,

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

$$\therefore \quad P = \frac{\pi^2 EI}{4l^2}$$

34.9. Columns with Both Ends Fixed

Consider a column AB of length l fixed at both of its ends A and B and carrying a critical load at B. As a result of loading, let the column deflect as shown in Fig. 34.4

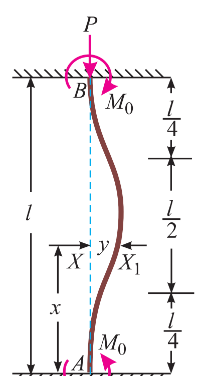


Fig. 34.4

Now consider any section X at a distance x from A.

Let P = Critical load on the column and

y = Deflection of the column at X.

A little consideration will show that since both the ends of the beam AB are fixed and it is carrying a load, therefore there will be some fixed end moments at A and B.

Let M_0 = Fixed end moments at A and B.

$\therefore$ Moment due to the critical load P,

$$M = -P \cdot y$$

$$EI\frac{d^2y}{dx^2} = M_0 - P \cdot y$$

...(Minus sign due to concavity initial centre line)

$$\therefore \quad \frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{M_0}{EI}$$

The general solution of the above differential equation is:

$$y = A\cos\left(x\sqrt{\frac{P}{EI}}\right) + B\sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \quad ...(i)$$

where A and B are the constants of integration. We know that when $x = 0$, then $y = 0$. Therefore $A = -\frac{M_0}{P}$. Now differentiating the above equation,

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + B\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right)$$

We also know that when $x = 0$, then $\frac{dy}{dx} = 0$. Therefore

$$0 = B\sqrt{\frac{P}{EI}}$$

A little consideration will show, that either B is equal to zero, or $\sqrt{\frac{P}{EI}}$ is equal to zero. Since the load P is not equal to zero, it is thus obvious that B is equal to zero. Substituting the values $A = \frac{M_0}{P}$ and $B = 0$ in equation (i),

$$y = -\frac{M_0}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} = \frac{M_0}{P}\left[1-\cos\left(l\sqrt{\frac{P}{EI}}\right)\right]$$

We also know that when $x = l$, then $y = 0$. Therefore

$$0 = \frac{M_0}{P}\left[1-\cos\left(l\sqrt{\frac{P}{EI}}\right)\right]$$

$$\therefore \quad \cos\left(l\sqrt{\frac{P}{EI}}\right) = 1$$

or

$$l\sqrt{\frac{P}{EI}} = 0 = 2\pi = 4\pi = 6\pi = ...$$

Now taking the least significant value,

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

$$\therefore \quad P = \frac{4\pi^2 EI}{l^2}$$

Alternative Methods

1. The fixed beam AB may bne considered as equivalent to a column of length $\frac{l}{2}$ with both ends hinged (*i.e.*, middle portion of the column as shown in Fig. 34.4).

$$\therefore \text{ Critical load,} \quad P = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}$$

2. The fixed beam AB may also be considered as equivalent to a column of length $\frac{l}{4}$ with one end fixed and the other free (*i.e.*, lower one-fourth portion of the beam as shown in Fig. 34.4).

$$\therefore \text{ Critical load,} \quad P = \frac{\pi^2 EI}{4\left(\frac{l}{4}\right)^2} = \frac{4\pi^2 EI}{l^2}$$

34.10. Columns with One End Fixed and the Other Hinged

Consider a column AB of length l fixed at A and hinged at B and carrying a critical load at B. As a result of loading, let the column deflect as shown in Fig. 34.5.

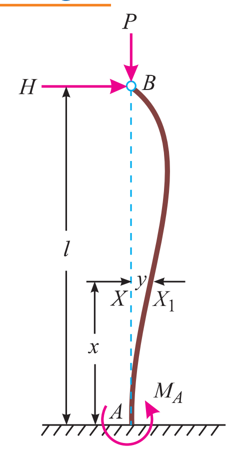


Fig. 34.5

Now consider any section X at a distance x from A.

Let P = Critical load on the column, and

y = Deflection of the beam at X,

A little consideration will show, that since the beam AB is fixed at A and it is carrying a load, therefore, there will be some fixed end moment at A. In order to balance the fixing moment at A, there will be a horizontal reaction at B.

Let M_A = Fixed end memeent at A and

H = Horizontal reaction at B.

∴ Moment due to critical load P,

$$M = -P \cdot y$$ ...(Minus sign due to conscavity towards initial centre line

or
$$EI \frac{d^2y}{dx^2} = H(l-x) - P \cdot y$$

∴
$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = \frac{H(l-x)}{EI}$$

The general solution of the above differential equation is

$$A = y \cos\left(x\sqrt{\frac{P}{EI}}\right) + B \sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{H(l-x)}{P} \qquad ...(i)$$

where A and B are the constants of integration. We know that when $x = 0$, they $y = 0$. Therefore $A = \frac{Hl}{P}$.

Now differentiating the above equation,

$$\frac{dy}{dx} = -A\sqrt{\frac{P}{EI}} \sin\left(x\sqrt{\frac{P}{EI}}\right) + B\sqrt{\frac{P}{EI}} \cos\left(x\sqrt{\frac{P}{EI}}\right) - \frac{H}{P}$$

We know that when $x = 0$, $\frac{dy}{dx} = 0$. Therefore

$$0 = B\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

∴
$$B = \frac{P}{H} \times \sqrt{\frac{EI}{P}}$$

We also know that when $x = l$, then $y = 0$. Therefore substituting these values of x, A and B is equation (i),

$$0 = \frac{Hl}{P}\cos\left(l\sqrt{\frac{P}{EI}}\right) + \frac{H}{P}\sqrt{\frac{EI}{P}}\sin\left(l\sqrt{\frac{P}{EI}}\right)$$

∴
$$\frac{H}{P}\sqrt{\frac{EI}{P}}\sin\left(l\sqrt{\frac{P}{EI}}\right) = \frac{Hl}{P}\cos\left(l\sqrt{\frac{P}{EI}}\right)$$

or $$\tan\left(l\sqrt{\frac{P}{EI}}\right) = \left(l\sqrt{\frac{P}{EI}}\right)$$

A little consideration will show that the value of $\left(l\sqrt{\frac{P}{EI}}\right)$ in radians, has to be such that its tangent is equal to itself. We know that the only angle, the value of whose tangent is equal to itself, is about 4.5 radians.

$$\therefore \quad l\sqrt{\frac{P}{EI}} = 4.5 \quad \text{or} \quad l^2 \times \frac{P}{EI} = 20.25 \quad \text{or} \quad P = \frac{20.25\, EI}{l^2}$$

$$\therefore \quad P = \frac{2\pi^2 EI}{l^2}$$

Note: A little consideration will show that 20.25 is not exactly equal to $2\pi^2$, but approximately equal to $2\pi^2$. This has been done to rationalise the value of P, *i.e.*, crippling load in various cases.

34.11. Euler's Formula and Equivalent length of a Column

In the previous articles, we have derived the relations for the crippliing load under various end conditions. Sometimes, all these cases are represented by a general equation called Euler's formula,

$$P_E = \frac{\pi^2 EI}{L_e^2}$$

where L_e is the equivalent or effective length of column.

The is another way of representing the equation, for the crippling load by an equivalent length of effective length of a column. The equivalent length of a given column with given end conditions, is the length of an equivalent column of the same material and cross-section with both ends hinged and having the value of the crippling load equal to that of the given column.

The equivalent lengths (L) for the given end conditions are given below:

Table 34.1

S.No.	*End conditions*	*Relation between equivalent length (L_e) and actual length (l)*	*Crippling load (P)*
1.	Both ends hinged	$L_e = l$	$P = \frac{\pi^2 EI}{(l)^2} = \frac{\pi^2 EI}{l^2}$
2.	One end fixed and the other free	$L_e = 2\,l$	$P = \frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}$
3.	Both ends fixed	$L_e = \frac{l}{2}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 EI}{l^2}$
4.	One end fixed and the other hinged	$L_e = \frac{l}{\sqrt{2}}$	$P = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI}{l^2}$

Note. The vertical column will have two moments of inertia (*viz.*, I_{XX} and L_{YY}). Since the column will tend to buckle in the direction of leas moment of inertia, therefore the least value of the two moments of inertia is to be used in tlhe relation.

34.12. Slenderness Ratio

We have already discussed in Art. 34.11 that the Euler's formula for the crippling load,

$$P_E = \frac{\pi^2 EI}{L_e^2} \quad \text{...}(i)$$

We know that the buckling of a column under the crippling load will take place about the axis of least resistance. Now substituting $I = Ak^2$ (where A is the area and k is the least radius of gyration of the section) in the above equation,

$$P_E = \frac{\pi^2 E\left(Ak^2\right)}{L_e^2} = \frac{\pi^2 EA}{\left(\frac{L_e}{k}\right)^2} \quad \text{...}(ii)$$

where $\frac{L_e}{k}$ is known as slenderness ratio. Thus slenderness ratio is defined as ratio of equivalent (or unsupported) length of column to the least radius of gyration of the section.

Slenderness ratio does not have any units.

Note. It may be noted that the formula for crippling load, in the pervious articles, have been derived on the assumption the the slenderness ratio $\frac{L_e}{k}$ is so large, that the failure of the column occurs only due to bending, the effect of direct stress (*i.e.*, $\frac{P}{A}$) being negligible.

34.13. Limitation of Euler's Formula

We have discussed in Art. 32.12 that the Euler's formula for the crippling load,

$$P_E = \frac{\pi^2 EA}{\left(\frac{L_e}{k}\right)^2}$$

∴ Euler's crippling stress,

$$\sigma_E = \frac{P}{A} = \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2}$$

A little consideration will show that the crippling stress will be high, when the slenderness ratio is small. We know that the crippling stress for a column cannot be more than the crushing stress of the column material. It is thus obvious that the Euler's formula will give the value of crippling stress of the column (equal to the crushing stress of the column material) corresponding to the slenderness ratio. Now consider a mild steel column. We know that the crushing stress for the mild steel is 320 MPa or 320 N/m^2 and Young's modulus for the mild steel is 200 GPa or 200×10^3 N/mm^2.

Now equating the crippling stress to the crushing stress,

$$320 = \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2} = \frac{\pi^2 \times (200 \times 10^3)}{\left(\frac{L_e}{k}\right)^2}$$

$$\therefore \qquad \left(\frac{L_e}{k}\right)^2 = \frac{\pi^2 \times 200 \times 10^3}{320}$$

or

$$\frac{L_e}{k} = 78.5 \text{ say } 80$$

Thus, if the slenderness ratio is less than 80 the Euler's formula for a mild steel column is not valid.

Sometimes, the columns, whose slenderness ratio is *more than* 80 are known as *long columns* and those whose slenderness ratio is *less than* 80 are known as short columns. It is thus obvious that the Euler's formula holds good only for long columns.

NOTE. In the Euler's formula, for crippling load, we have not taken into account the direct stresses induced in the material due to the load, (which increases gradually from zero to its crippling value). As a matter of fact, the combined stress, due to direct load and slight bending reaches its allowable value at a load, lower than that required for buckling ; and therefore this will be the limiting value of the safe load.

EXAMPLE 34.1. *A steel rod 5 m long and of 40 mm diameter is used as a column, with on end fixed and the other free. Determine the crippling load by Euler's formula. Take E as 200 GPa.*

SOLUTION. Given : Length (l) = 5 m = 5×10^3 mm ; Diameter of column (d) = 40 mm and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

We know that moment of iertia of the column section,

$$I = \frac{\pi}{64} \times (d)^4 = \frac{\pi}{64} \times (40)^4 = 40\,000\,\pi \text{ mm}^4$$

Since the column is fixed at one end and free at the other, therefore equivalent length of the column,

$$L_e = 2\,l = 2 \times (5 \times 10^3) = 10 \times 10^3 \text{ mm}$$

$$\therefore \text{ Euler's crippling load,} \quad P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times (200 \times 10^3) \times (40000\,\pi)}{(10 \times 10^3)^2} = 2480 \text{ N}$$

$$= 2.48 \text{ kN} \qquad \textbf{Ans.}$$

EXAMPLE 34.2. *A hollow alloy tube 4 m long with external and internal diameters of 40 mm and 25 mm respectively was found to extend 4.8 mm under a tensile load of 60 kN. Find the buckling load for the tube with both ends pinned. Also find the safe load on the tube, taking a factor of safety as 5.*

SOLUTION. Given : Length l = 4 m ; External diameter of column (D) = 40 mm ; Internal diameter of column (d) = 25 mm ; Deflection (δl) = 4.8 mm ; Tensile load = 60 kN = 60×10^3 N and factor of safety = 5.

Buckling load for the tube

We know that area of the tube,

$$A = \frac{\pi}{4} \times [D^2 - d^2] = \frac{\pi}{4}[(40)^2 - (25)^2] = 765.8 \text{ mm}^2$$

and moment of inertia of the tube,

$$I = \frac{\pi}{64}[D^4 = d^4] = \frac{\pi}{64}[(40)^4 - (25)^4] = 106\,500 \text{ mm}^4$$

We also know that strain in the alloy tube,

$$e = \frac{\delta l}{l} = \frac{4.8}{4 \times 10^3} = 0.0012$$

and modulus of elasticity for the alloy,

$$E = \frac{\text{Load}}{\text{Area} \times \text{Strain}} = \frac{60 \times 10^3}{765.8 \times 0.0012} = 65\ 290 \text{ N/mm}^2$$

Since the column is pinned at its both ends, therefore equivalent length of the column,

$$L_e = l = 4 \times 10^3 \text{ mm}$$

$$\therefore \quad \text{Euler's buckling load, } P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 65\,290 \times 106\,500}{(4 \times 10^3)^2} = 4290 \text{ N}$$

$$= 4.29 \text{ kN} \quad \textbf{Ans.}$$

Safe load for the tube

We also know that safe load for the tube

$$= \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{4.29}{5} = 0.858 \text{ kN} \quad \textbf{Ans.}$$

EXAMPLE 34.3. *Compare the ratio of the strength of a solid steel column to that of a hollow of the same cross-sectional area. The internal diameter of the hollow column is 3/4 of the external diameter. Both the columns have the same length and are pinned at both ends.*

SOLUTION. Give : Area of solid steel colum $A_S = A_H$ (where A_H = Area of hollow column) ; Internal diameter of hollow column (d) = 3 D/4 (where D = External diameter) and length of solid column (l_S) = l_H (where l_H = Length of hollow column).

Let D_1 = Diameter of the solid column,

k_H = Radius of gyration for hollow column and

k_S = Radius of gyration for solid column.

Since both the columns are pinned at their both ends, therefore equivalent length of the solid column,

$$L_S = l_S = L_H = l_H = L$$

We know that Euler's crippling load for the solid column,

$$P_S = \frac{\pi^2 EI}{L_H^2} = \frac{\pi^2 E \cdot A_S \cdot k_S^2}{L^2} \qquad ...(i)$$

Similarly Euler's crippling load for the hollow column

$$P_H = \frac{\pi^2 EI}{L_H^2} = \frac{\pi^2 E \cdot A_H \cdot k_H^2}{L^2} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*),

$$\frac{P_H}{P_S} = \left(\frac{k_H}{k_S}\right)^2 = \frac{\frac{D^2 + d^2}{16}}{\frac{D_1^2}{16}} = \frac{D^2 + d^2}{D_1^2} = \frac{D^2 = \left(\frac{3D}{4}\right)^2}{D_1^2}$$

$$= \frac{25\, D^2}{16\, D_1^2} \qquad ...(iii)$$

Since the cross-sectional areas of the both the columns is equal, therefore

$$\frac{\pi}{4} \times D_1^2 = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[D^2 - \left(\frac{3D}{4}\right)^2\right] = \frac{\pi}{4} \times \frac{7\, D^2}{16}$$

$$\therefore \qquad D_1^2 = \frac{7D^2}{16}$$

Now substituting the value of D_1^2 in equation (*iii*),

$$\frac{P_H}{P_S} = \frac{25D^2}{16 \times \frac{7D^2}{16}} = \frac{25}{7} \qquad \textbf{Ans.}$$

EXAMPLE 34.4. *An I section joist 400 mm × 200 mm × 20 mm and 6 m long is used as a strut with both ends fixed. What is Euler's cripp;ing load for the column? Take Young's modulus for the joist as 200 GPa.*

SOLUTION. Given : Outer depth (D) = 400 mm ; Outer width (B) = 200 mm ; Length (l) = 6 m = 6×10^3 mm and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

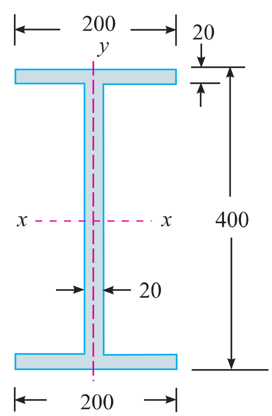


Fig. 34.6

From the geometry of the figure, we find that inner depth,

$$d = 400 - (2 \times 20) = 360 \text{ mm}$$

and inner width, $\qquad b = 200 - 20 = 180$ mm

We know that moment of inertia of the joist section about X-X axis,

$$I_{XX} = \frac{1}{12}[BD^2 - ba^3]$$

$$= \frac{1}{12}[200 \times (400)^3 - 180 \times (360)^3]\,\text{mm}^4$$

$$= 366.8 \times 10^6 \text{ mm}^4 \qquad ...(i)$$

Similarly, $$I_{YY} = \left[2 \times \frac{2 \times (200)^3}{12}\right] + \frac{360 \times (20)^3}{12}\text{mm}^4$$

$$= 2.91 \times 10^6 \text{ mm}^4 \qquad ...(ii)$$

Since I_{YY} is less than I_{XX}, therefore the joist will tend to buckle in Y-Y direction. Thus, we shall take the value of I as $I_{YY} = 2.91 \times 10^6$ mm^4. Moreover, as the column is fixed at its both ends, therefore equivalent length of the column,

$$L_e = \frac{l}{2} = \frac{(6 \times 10^3)}{2} = 3 \times 10^3 \text{ mm}$$

∴ Euler's crippling load for the column,

$$P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times (200 \times 10^3) \times (2.91 \times 10^6)}{(3 \times 10^3)^2} = 638.2 \times 10^3 \text{ N}$$

$$= 638.2 \text{ kN} \qquad \textbf{Ans.}$$

EXAMPLE 34.5. *A T-section 150 mm × 120 mm × 20 mm is used as a strut of 4 m long with hinged at its both ends. Calculate the crippling load, if Young's modulus for the material be 200 GPa.*

SOLUTION. Given : Size of T-section = 150 mm × 120 mm × 20 mm ; Length (l) = 4 m = 4×10^3 mm and Young's modulus (E) = 200 GPa = 200×10^3 N/mm^2.

First of all, let us find the centre of the T-section; Let bottom of the web be the axis of reference.

Web

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_1 = \frac{100}{2} = 50 \text{ mm}$$

Flange

$$a_2 = 150 \times 20 = 3000 \text{ mm}^2$$

$$y_2 = 120 - \left(\frac{20}{2}\right) = 110 \text{ mm}$$

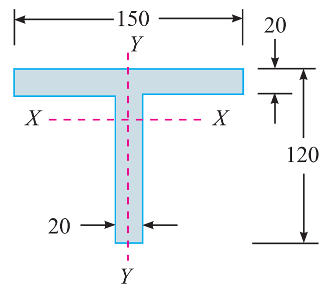


Fig. 34.7

We know that distance between the centre of gravity of the T-section and bottom of the web

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (3000 \times 110)}{200 + 3000} = 86 \text{ mm}$$

We also know that moment of inertia of the *T*-section about *X-X* axis,

$$I_{XX} = \left(\frac{20 \times (100)^3}{12} + 2000 \times (36)^2\right) + \left(\frac{150 \times (20)^3}{12} + 3000 \times (24)^2\right) \text{mm}^4$$

$$= (4.26 \times 10^6) + (1.83 \times 10^6) = 6.09 \times 10^6 \text{ mm}^4$$

Similarly, $$I_{YY} = \frac{100 \times 9200)^3}{12} + \frac{20 \times (150)^3}{12} = 5.069 \times 10^6 \text{ mm}^4$$

Since I_{YY} is less than I_{XX}, therefore the column will tend to buckle in *Y-Y* direction. Thus, we shall take the value of I as $I_{YY} = 5.69 \times 10^6$ mm^4. Moreover, as the column is hinged at its both ends, therfore length of the column,

$$L_e = l = 4 \times 10^3 \text{ mm}$$

∴ Euler's crippling load, $$P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times (200 \times 10^3) \times (5.69 \times 10^6)}{(4 \times 10^3)^2} = 702 \times 10^3 \text{ N}$$

$$= 702 \text{ kN}$$ **Ans.**

EXERCISE 34.1

1. A mild steel column of 50 mm diameter is hinged at both of its ends. Find the crippling load for the column, if its length is 2.5 m. Take *E* for the column material as 200 GPa. [**Ans.** 96.9 kN]
2. A hollow cast iron column of 150 mm external diameter and 100 mm internal diameter is 3.5 m long. If one and of the column is rigidity fixed and the other is free, find the critical load on the column. Assume modulus of elasticity for the column material as 120 GPa. [**Ans.** 482 kN
3. A 1.75 m long steel column of rectangular cross-section 120 mm × 100 mm is rigidity fixed at one end and hinged at the other. Determine the buckling load on the column and the corresponding axial stress using Euler's formula. Take *E* for the column material as 200 GPa. [**Ans.** 12.84 MN ; 1070 MPa]
4. An -section 240 mm × 120 mm × 20 mm is used as 6 m long column with both ends fixed. What is the crippling load for the column? Take Young's modulus for the joist as 200 GPa. [**Ans.** 1292.5 kN]