GEOMETRIC DESIGN OF HIGHWAYS

Geometric design for transportation facilities includes the design of geometric cross sections, horizontal alignment, vertical alignment, intersections and various design details. These basic elements are common to all linear facilities, such as roadways, railways, and airport runways and taxiways. Although the details of design standards vary with mode and class of facility, most of the issues involved in geometric design are similar for all modes. In all cases, the goals of geometric design are to maximize the comfort, safety and economy of the facilities, while maximizing their environmental impacts. We therefore focus on the fundamentals of geometric design, and standards and examples from different modes.

Basic physical elements of a highway

The basic features of a highway are the carriageway itself, expressed in terms of the number of lanes used, the central reservation or median strip, the shoulders (including verges) and drainage. Depending on the level of the highway relative to the surrounding terrain, side-slopes may also be a design issue.

Main carriageway

The chosen carriageway depends on a number of factors, most notably the volume of traffic using the highway, the quality of service expected from the installation and the selected design speed. In most situations a lane width of 3.65 m is used, making a standard divided or undivided 2-lane carriageway 7.3 m wide in total. Table 1 gives a summary of carriageway widths normally used in the UK. These widths are as stated in TD 27/96 (DoT, 1996). Any reduction or increase in these widths is considered a departure from standard. The stated lane widths should only be departed from in exceptional circumstances such as where cyclists need to be accommodated or where the number of lanes needs to be maximized for the amount of land available. In Scotland and Northern Ireland, a total carriageway width of 6.0m may be used on single carriageway all-purpose roads where daily flow in the design year is estimated not to exceed 5000 vehicles.

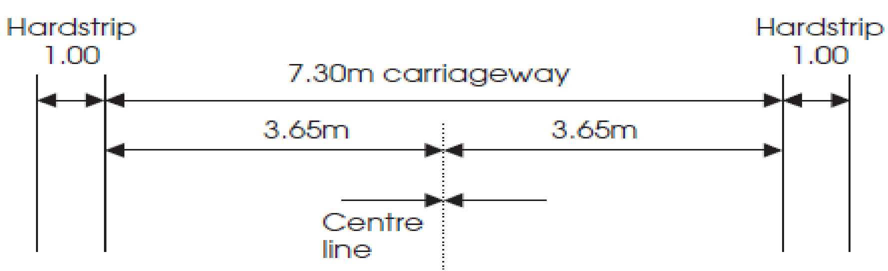
Road description	Carriageway width (m)
Urban/rural 4-lane dual	14.60
Urban/rural 3-lane dual	11.00
Urban/rural single/dual 2-lane (normal)	7.30
Rural single 2-lane (wide)	10.00

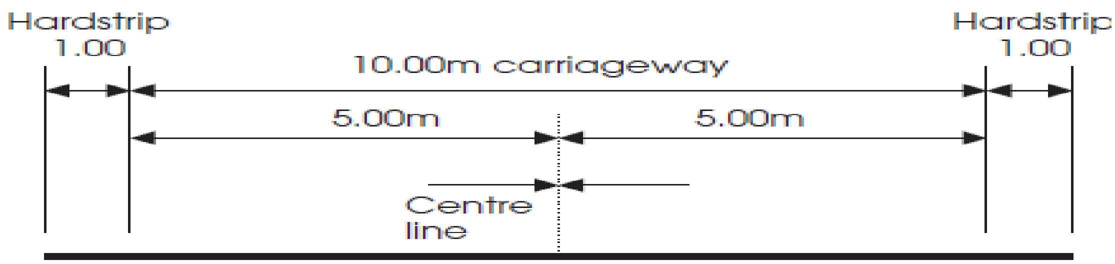
Central reservation

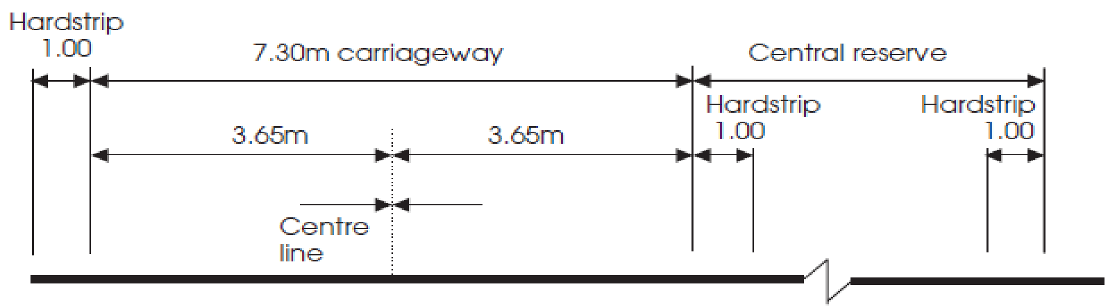
A median strip or central reservation divides all motorways/dual carriageways. Its main function is to make driving safer for the motorist by limiting locations where vehicles can turn left (on dual carriageways), completely separating the traffic travelling in opposing directions and providing a space where vehicles can recover their position if for some reason they have unintentionally left the carriageway. In urban settings, a width of 4.5 m is recommended for 2/3-lane dual carriageways, with 4.0m recommended for rural highways of this type. While these values should be the first option, a need to minimize land take might lead to reductions in their value. Use of dimensions less than those recommended is taken as a relaxation rather than a departure from the standard (TD27/96). (The term ‘relaxation’ refers to a relaxing of the design standard to a lower level design step, while a ‘departure’ constitutes non-adherence to a design standard where it is not realistically achievable. Use of central reservation widths greater than the values stated is permitted. Its surfacing material should be different to that on the carriageway itself. Grass, concrete or bituminous material can be used.

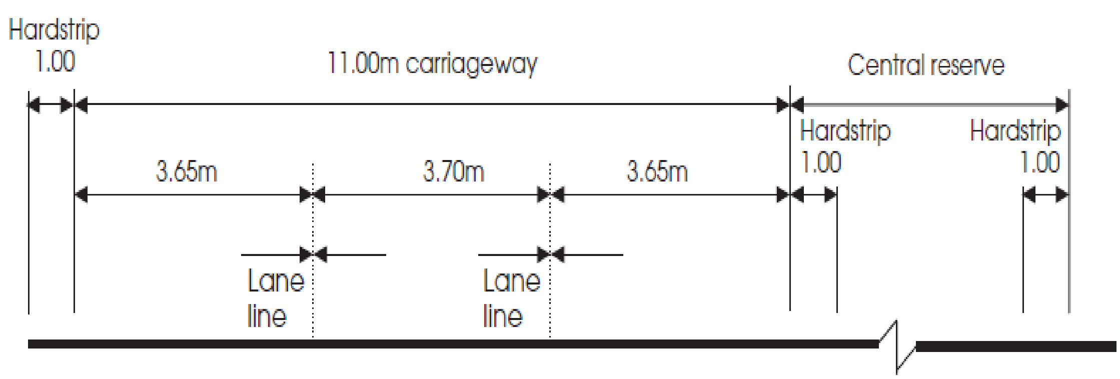
Hard strips/verges

On single carriageway roads (normal and wide), a 1m wide hardstrip and a 2.5 m wide grassed verge is employed on the section of roadway immediately adjacent to the main carriageway on each side. On rural 2 and 3-lane motorways, a hardshoulder of 3.3 m and a verge of 1.5 m are the recommended standard. On rural 2/3-lane dual carriageways, the 1m wide hardstrip and 2.5 m wide verge is detailed on the nearside with a 1m hardstrip on the offside. For urban motorways the verge dimension varies while the hard shoulder is set at 2.75m wide. Diagrams of typical cross-sections for different road classifications are given in Figs 6.1 to 6.4.









The proper geometric design of a highway ensures that drivers use the facility with safety and comfort. The process achieves this by selecting appropriate vertical and horizontal curvature along with physical features of the road such as sight distances and superelevation. The ultimate aim of the procedure is a highway that is both justifiable in economic terms and appropriate to the local environment.

Highway cross sections consists of travelled way, shoulders (or parking lanes), and drainage channels. Shoulders are intended primarily as a safety feature. They provide for accommodation of stopped vehicles, emergency use, and lateral support of the pavement.

Shoulders may either be paved or unpaved. Drainage channels may consist of ditches (usually grass swales) or of paved shoulders with berms or curbs and gutters.
Standard lane with is 3.65 m, although narrower lanes are common on older roadways, and may still be provided in cases where the standard lane width is not economical.

Vertical alignment

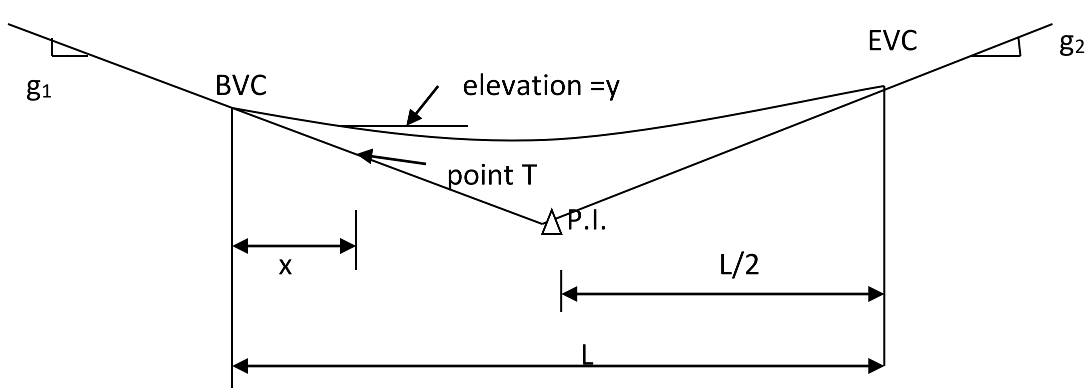
The vertical alignment of a transportation facility consists of tangent grades (straight lines in the vertical plane) and vertical curves. Vertical alignment is documented by the profile. The profile is a graph that has elevation as its vertical axis and distance, measured in stations along the centerline or other horizontal reference line of the facility, as horizontal axis.

Tangent grades

Tangent grades are designated according to their slopes or grades. Maximum grades vary, depending on the type of facility, and usually do not constitute an absolute standard. The effect of steep grade is to slow down heavier vehicles (which typically have the lowest power/weight ratios) and increase operating costs. Furthermore, the extent to which any vehicle (with a given power/ weight ratio) is slowed depends on both the steepness and length of the grade. The effect of slowing down the heavier vehicles depends on the situation, and is often more a matter of traffic analysis than simple geometric design. As result the maximum grade for a given facility is a matter of judgment, with the tradeoffs usually being cost of the construction versus speed.

Vertical curves

Vertical tangents with different grades are joined by vertical curves such as the one shown in figure below, it is a symmetric vertical curve.



Vertical curves are normally parabolas centred about the point of intersection (P.I) of the vertical tangent they join. Vertical curves are thus of the form

$y = y_o + g_1 x + \frac{r^2}{2}$ ---------------- (1)

Where y = elevation of a point on the curve

y_o= elevation of the beginning of the vertical curve (BVC)

g_1 = grade just prior to the curve

x = horizontal distance from the BVC to the point on the curve

r = rate of change of grade

the rate of change of grade, in turn, is given by

$r = \frac{g_1 - g_2}{L}$ ………………………….(2)

Where g_2 is the grade just beyond the end of the vertical curve (EVC) and L is the length of the curve. Also, vertical curves are sometimes described by K, the reciprocal of r. K is the distance in metres required to achieve a 1 % change in grade. Vertical curves are classified as sags where g2 >g_1 and crest otherwise. Note that r (and hence the term$\frac{rx^2}{2}$) will be positive for sags and negative for crests.

Also note that the vertical distances in the vertical curve formulas are product of grade times a horizontal distance. In consistent units, if vertical distances are to be given in metres, horizontal distances should also be in metres and grades should be dimensionless ratios. In many cases, however, it is more convenient to represent grades in percent and horizontal distances in stations. This produces the correct result because the grade is multiplied by 100 and the horizontal distance divided by 100, and the two factors of 100 cancel. It is very important not to mix the two methods, however. If grades are in percent, horizontal distances must be in stations; likewise, if grades are dimensionless ratios, horizontal distances must be in metres. The parabola is selected as the vertical curve so that the rate of change of grade, which is the second derivative of the curve, will be constant with distance. Note that the first derivative is the grade itself, and since the rate of change is constant, the grade of any point in the vertical curve is a linear function of the distance from the BVC to the point. That is,

$$g = \frac{\partial y}{\partial x} = g_1 \ + rx - - - - - - - (3)$$

The quantity $\frac{rx^2}{2}$ is the distance from the tangent to the curve and is known as the offset. If x is always measured from the BVC, the offset given by $\frac{rx^2}{2}$ will be measured from g_1 tangent. To determine the offset from the g_2 tangent, x should be measured backwards from the EVC. Since the curve is symmetrical about its centre, the offsets from the g_1 and g_2 tangents, respectively, are also symmetrical about the centre of the curve which occurs at the station of its P.I.

Other properties of the vertical curve may be used to sketch it. For instance, at its centre, the curve passes halfway between the P.I and a chord joining the BVC and EVC. At the quarter points, it passes one quarter of the way between the tangents and the chord. Normal drafting practice is to show the P.I by means of a triangular symbol, although the extended vertical tangents shown in the figure are often omitted. The BVC and EVC are shown by means of circular symbols. The P.I., BVC, EVC are identified by notes. The stations of the BVC and EVC are given notes, as are the stations and elevation of the P.I., the two tangent grades, and the length of the vertical curve.
Elevations on vertical curves are easily calculated by means of a calculator, computer, or spreadsheet program. On traditional way of representing then is in the form of a table shown below. The table represents a 300 m sag vertical curve between a + 1.0 % grade and a +6.0% grade.

Station	Grade	Tangent elevation	Offset	Profile elevation
99 +75	+1%	149.75		149.75
100 +00	BVC	150.00		150.00
100+25		150.25	0.05	150.30
100+50		150.50	+0.21	150.71
100+75		150.75	+0.47	151.22
101+00		151.00	+0.83	151.83
101+25		151.25	+1.30	152.55
101+50	P.I	151.50	+1.88	153.38
101+75		153.00	+1.30	154.30
102+00		154.50	+0.83	155.33
102+25		156.00	+0.47	156.47
102+50		157.50	+0.21	157.71
102+75		159.00	+0.05	159.00
103+00	EVC	160.50		160.50
103+25	6%	162.00		162.00

The first column gives the station. The second column gives the intersecting grades and the locations of the BVC, P.I., and EVC. The third column gives the elevation of each point on the tangent grades, calculated as BVC elevation plus g_1x for the first tangent grade and P.I. elevation plus $g_2(x-L/2)$for the second. The fourth column gives the offset, calculated as $\frac{rx^2}{2}$ with x measured from either BVC or EVC as appropriate: since the offsets are symmetrical about the P.I., however they need to be calculated only from the BVC to the P.I. the last column gives the curve elevation, which is the tangent elevation plus the offset. It should be noted that curve elevations can also be calculated by using only offsets from the g_1 tangent, and that in many cases it may be more convenient to use only one tangent.

Example

A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I station is 100 + 00 and the P.I elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Solution

Rate of change of grade:

$r = \frac{g_1 - g_2}{L} = \frac{1.0\% - (-2.5\%)}{1.8\ sta} = 1.944\%/sta$

Station of the low point

At low point, g =0

g= g_1 + rx =0

or

$$x = \frac{-g_1}{r} = -\left(\frac{-2.5}{1.944}\right) = 1.29 = 1 + 29\ sta$$

Station of BVC =(100 +00) –(0+90) =99+10

Station of low point = (99+ 10) + (1+29) =100 +39

Elevation of BVC:

$$y_{0=}100.0\ m + (-0.9\ sta)(-2.5\%) = \ 102.25\ m$$

Elevation of low point:

y =y_o+g_1x + $\frac{r^2}{2}$

$$= 102.25\ m + (-2.5\%)(1.29\ sta) + \frac{(1.944\%\ sta)(1.29\ sta)^2}{2}$$

= 100.64 m

Design standards for vertical curves establish their minimum lengths for specific circumstances. For highways, minimum length of vertical curve may be based on sight distance, on comfort standards involving vertical acceleration, or appearance criteria. For airport runways and taxiways, minimum vertical curve lengths are based on sight distance.
In most cases, sight distance or appearance standards will gorvern for highways. The equations used to calculate minimum length of vertical curves based on the sight distance depend on whether the sight distance is greater than or less than the vertical curve length. For crest vertical curves, the minimum length depends on the sight distance, the height of the driver's eye and the height of the object to be seen over the crest of the curve. The minimum length is given by the formula

$$L_{min} = \frac{AS^2}{200(\sqrt{h_1}+\sqrt{h_2})^2} \quad when\ S \leq L \text{..........................(4a)}$$

$$L_{min} = 2S - \frac{200(\sqrt{h_1}+\sqrt{h_2})^2}{A} \quad when\ S \geq L \text{ ....(4b)}$$

Where S = sight distance

L = vertical curve length

A = absolute value of the algebraic difference in grades, in percent $|g_1 - g_2|$

h_1 = height of eye

h_2 = height of object

For stopping sight distance, the height of the object is normally taken to be 0.150 m. For passing sight distance, the height of object used by AASHTO is 1.300 m. height of eye is assumed to be 1.070 m.
Inserting these standard values for h_1 and h_2, equation 4 may be reduced to

$$L_{min} = \frac{AS^2}{404} \quad when\ S \leq L$$

$$= 2S - \frac{404}{A} \quad when\ s \geq L$$

for stopping sight distance and

$$L_{min} = \frac{AS^2}{946} \quad when\ S \leq L$$

$$= 2S - \frac{946}{A} \quad when\ s \geq L$$

for passing sight distance.

For sag vertical curves, stopping sight distance is based on the distance illuminated by the headlights at night. Design standards are based on an assumed headlight height of 0.600 m and an upward divergence of the headlight beam of 1^0. As the case of the crest vertical curves, the formulas for minimum length of vertical curve depend on whether the length of the curve is greater or less than the sight distance. For sag vertical curves, the formula is

$$L_{min} = \frac{AS^2}{200[0.6+S(\tan 1^0)]} = \frac{AS^2}{120+3.5S} \quad when\ S \leq L$$

$$2S - \frac{200[0.6+S(\tan 1^0)]}{A} = 2S - \frac{120+3.5S}{A}\ when\ S \geq L$$

Design charts or tables are used to determine minimum length of vertical curve to provide stopping sight distance for both crest and sag vertical curves, and passing sight distance on crests. These may be found in the AASHTO Policy on Geometric Design of Highways and Streets. In some cases, sag vertical curves with a small total grade change can be sharp enough to cause discomfort without violating sight distance standards. In this case, it is necessary to establish a comfort criterion of the form

$$r \leq \frac{a}{v^2}$$

Where r is the rate of change of grade, a is the maximum radial acceleration permitted, and v is the speed. There is no general agreement as to the maximum value of radial acceleration that can be tolerated without producing discomfort. AASHTO suggests a value of 0.3 m/s^2, and suggests the standard

$$L \geq \frac{AV^2}{395}$$

Where L = length of vertical curve, m

A = g_2- g_1, percent

V = design speed, km/h

Minimum vertical curve standards for highways may also be based on appearance. This problem arises because short vertical curves tend to look like kinks when viewed from a distance. Appearance standards vary from agency to agency. Current California standards for instance, require a minimum vertical curve length of 60 m where grade breaks are less than 2% or design speeds are less than 60 km/h, the minimum vertical curve length is given by L = 2V, where L in the vertical curve length in metres and V is the design speed in Km/h.

Example

Determine the minimum length of a crest vertical curve between a +0.5% grade and a -1.0% grade for a road with 100 km/h design speed. The vertical curve must provide 190 m stopping distance and meet the California appearance stopping distance criterion

Solution

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{200(\sqrt{h_1}+\sqrt{h_2})^2} = \frac{[0.5-(-1.0)](190^2)}{200(\sqrt{1.017}+\sqrt{0.150})^2} = 134.0\ m$$

134.0 m <190 m, so S > L

$$L = 2S - \frac{200(\sqrt{h_1}+\sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070}+\sqrt{0.150})^2}{[0.5-(-1.0)}$$

= 380.0 – 269.5 = 110.5 m

Appearance criterion

Design speed = 100km/h > 60 km/h but grade break = 1.5% < 2%. Use 60 m.

Conclusion:

Sight distance criterion governs. Use 120 m vertical curve.

Example

Determine the minimum length of a sag vertical curve between a -0.7% and a + 0.5% grade for a road with a 110 km/h design speed. The vertical curve must provide 220 m stopping sight distance and meet the California appearance criteria and the AASHTO comfort standard. Round up to the next greatest 20 m interval.

Assume $S \leq L$

$$L = \frac{AS^2}{120+3.5S} = \frac{[0.5-(-0.7)](220)^2}{120+3.5\ (220)} = 65.3\ m$$

65.3 m < 220 m, so S > L

$$L = 2S - \frac{120+3.5S}{A} = 2(220) - \frac{120+3.5(220)}{[0.5-(-0.7)]}$$

= 440 -741.7 = 301.7 m

Since L< 0, no vertical curve is needed to provide stopping sight distance

Comfort criterion:

$$L = \frac{AV^2}{395} = \frac{[0.5-(-0.7)](110^2)}{395} = 36.8\ m$$

Appearance criterion:

Design speed = 110 km/h > 60 km/h but grade break = 1.2% < 2%. Use 60 m.

Conclusion:

Appearance criterion governs. Use 60 m vertical curve.

Finally, vertical curve lengths may be limited by the need to provide clearances over or under objects such as overpasses or drainage structures. In the case of sag vertical curves passing over objects or crest vertical curves passing under them, the required clearances establish minimum lengths; in the case of crest vertical curves passing over objects or sags passing under them, the clearances exhibit maximum lengths. Where clearances limit vertical curve lengths, adequate sight distance should still be provided.

In either case, the maximum or minimum length of the vertical curve may be determined by assuming the clearance is barely met and calculating the length of the vertical curve passing through the critical point thus established. It is easiest to do this in the figure shown below. In the figure, C represents the critical clearance, z the horizontal distance from P.I. to the critical point, and y' the offset between the critical point and the tangent passing through the BVC.

The equation for the offset is

$$y' = \frac{rx^2}{2} \dots\dots\dots\dots\dots\dots 1$$

Where r, as before, is

$$r = \frac{g_2 - g_1}{L} = \frac{A}{L} \dots\dots\dots\dots\dots 2$$

And

$$x = \frac{L}{2} + z \dots\dots\dots\dots\dots\dots\dots 3$$

Substituting equation 2 & 3 into equation 1

$$y' = \frac{A(L/2 + z)^2}{2L} \dots\dots\dots\dots\dots 4$$

Expansion and rearrangement of equation 2 leads to the quadratic equation

$$AL^2 + (4Az - 8y')L + 4Az^2 = 0 \dots\dots\dots\dots\dots\dots 5$$

Solving equation 5 results in two roots. The smaller of these represents a vertical curve that is tangent between the P.I. and the critical point. Discarding the solution and letting w = y'/A to simplify the notation, the solution for L in the larger root leads to

$$L = 4w - 2z + 4\sqrt{(w^2 - wz)}$$

As an expression for the maximum or minimum vertical curve length.

Example

A vertical curve joins a – 1.2% grade to a + 0.8 % grade. The P.I. of the vertical curve is at station 75 + 00 and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station 75 + 40 by 0.80 m. the elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?

Determine z:

$$z = (75 + 40) - (75 + 00) = 0.40\ sta.$$

Determine y’

Elevation of tangent =$50.90 + (-1.2)(0.4) = 50.42\ m$

Elevation of roadway = $51.10 + 0.80 = 51.90\ m$

Y’ = 51.90 -50.42 =1.48 m

Determine w:

$A = g_2 - g_1 = (+0.8) - -1.2 = 2.0$

$w = \frac{y'}{A} = \frac{1.48}{2} = 0.74$

Determine L

$L = 4w - 2z + 4\sqrt{(w^2} - wz)$

$= 4(0.74) - 2(0.4) + \sqrt{[0.74^2} - (0.74)(0.4)] = 4.17sta = 417\ m$

Check y’

$x = \frac{4.17}{2} + 0.4 = 2.485\ sta$

$r = \frac{A}{L} = \frac{2}{4.17} = 0.48$

$y' = \frac{rx^2}{2} = \frac{(0.48)(2.485)^2}{2} = 1.48\ \ check$

Horizontal alignment

Horizontal alignment for linear transportation facilities such as highways and railways consists of horizontal tangents, circular curves and possibly transition curves. In the case of highways, transition curves are not always used.

Curves are generally used on highways and railways where it is necessary to change the direction of motion. They are employed to effect the gradual change of direction at the intersection of straight lines. The lines connected by the curves are tangential and are called tangents or straights. The curves are generally circular arcs but parabolic or spiral arcs are also in use.

Horizontal curves can be circular or non – circular(transitional)

Example

It is required to connect two straights whose deflection angle is $13^0\ 16'00''$ by a circular curve of radius 600 m. make the necessary calculations for setting out the curve by the tangented angle method if the through chainage of the intersection point is 2745.72 m. use a chord length of 25 m and sub – chord at the beginning and end of the curve to ensure that the pegs are placed at exact 25 m multiple of through chainage.

Tangent length = $R\tan\frac{\theta}{2} = 600\tan\left(\frac{13^0 16' 00''}{2}\right) = 69.78\ m$
Therefore chainage of $T_1 = 2745.72 - 69.78 = 2675.94\ m$
Round this figure to 2700 m
Length of initial sub chord = 2700 -2675.94 = 24.06 m
Length of circular curve = $\frac{R\theta\pi}{180} = \frac{600x\ 13^0 16' 00'' x\ \pi}{180} = 138.93\ m$
Chainage of $T_2\ = 2675.94 + 138.93 = 2814.87\ m$
Length of final sub chord
$= 2814.87 - 2800 = 14.87\ m$
The tangential angles for these chords are obtained from the formula
$\alpha = 1718.9\ x\ \left(\frac{chord\ length}{radius}\right) min$

Points	Chainage (m)	Chord length(m)	Individual tangential angle	Cumulative tangential angle
T_1	2675.94	0	$00^0 00' 00''$	$00^0 00' 00''$
C_1	2700.00	24.06	$01^0 08' 56''$	$01^0 08' 56''$
C_2	2725.00	25.00	$01^0 11' 37''$	$02^0 20' 33''$
C_3	2750.00	25.00		
C_4	2775.00	25.00		
C_5	2800.00	25.00		
T_2	2814.87	14.87	$00' 42' 36''$	$06^0 38' 00''$
	138.93 (check)			

Superelevation

The purpose of superelevation or banking of curves is to counteract the centrifugal force produced as a vehicle rounds a curve. The term itself comes from railroad practice, where the top of the rail is the profile grade. In curves, the profile grade line follows the lower rail, and the upper rail is said to be superelevated. Since most railways are built to a standard guage, the superelevations are given as the difference between in elevation between the upper and lower rail. In the case of the highway, more complicated modifications of the cross section are required, and because widths vary, superelevation is expressed as a slope.
A vehichle travelling on a horizontal curve exerts an outward force called centrifugal force. To resist this force and maintain the desired design speed, highway curves need to be superelevated. Superelevation may be defined as the rotation of the roadway cross section in such a manner as to overcome the centrifugal force that acts on a motor vehicle traversing a curve.

On the superelevated highway, the centrifugal force can be resisted by:

1. The weight component of the vehicle parallel to the superelevated surface
2. The side friction between the tires and the pavement
3. Introduction of transition curves
4. Pavement widening.

It is impossible to balance centrifugal force by superelevation alone, because for any given curve radius, a certain superelevation rate is exactly correct for only one operating speed around the curve. At all other speeds, there will be a side thrust outward or inward relative to the curve centre which must be offset by side friction

The transitional rate of applying superelevation into and out of curves is influenced by design speed, degree of curvature and number of lanes. Introducing superelevation permits a vehicle to travel through a curve more safely and at a higher speed than would be possible with a normal crown section. For a given degree of curvature, a steeper superelevation is required for a higher design speed than is needed for a lower design speed. For a given design speed more superelevation is needed through sharp curves than for relatively flat curves. The maximum rates of superelevation used on roadways are controlled by four factors:

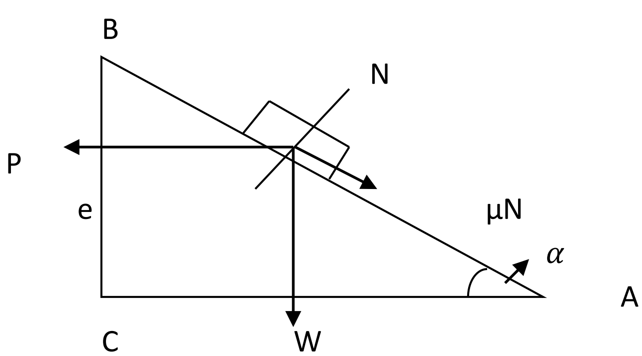
(1) Climate conditions (i.e. frequency of ice and snow);

(2) Terrain conditions (i.e. flat or rolling);

(3) Type of area (i.e. rural or urban); and

(4) Frequency of slow-moving vehicles.

Analysis of superelevation



$$P = \frac{WV^2}{gr}$$

Let the force acting on the vehicle while moving on a circular curve is as shown above. Here,
A = inner edge of the road
B = outer edge of the road
α = inclination of the road surface to horizontal
e = superelevation of road i.e raising of outer edge at rate of 1 horizontal to e vertical.

Therefore, $\tan\alpha = \frac{e}{1}$
W = weight of the vehicle acting vertically downwards
V = speed of vehicle in km/h
v = speed of vehicle at curve in metres/sec. = 0.25V
R = radius of curvature
μ = lateral coefficient of friction between the road surface and tyres
N = load reaction on the road surface
P = centrifugal force acting in outward direction or lateral force

$$P = \frac{WV^2}{gR}$$

Where

$$\frac{P}{W} = \frac{V^2}{gR}; \frac{P}{W} = 0.21\ to\ 0.25\ for\ roads\ and\ 0.125\ for\ railways$$

Resolving

1. Forces parallel to the road surfaces

$$\mu N + Wsin\alpha = \frac{Wv^2}{gR}cos\alpha \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (i)$$

2. Forces perpendicular to road surfaces

$$\frac{Wv^2}{gR}\sin\alpha + W\cos\alpha = N \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(ii)$$

Putting the value of R from equation (ii) to equation (i) we get

$$\mu\left(\frac{Wv^2}{gR}sin\alpha + Wcos\alpha\right) + \ Wsin\alpha = \frac{Wv^2}{gRcos\alpha}$$

Or

$$\frac{wv^2}{gR}sin\alpha + sin\alpha = \frac{v^2}{gR} - cos\alpha$$

Or

$$sin\alpha\left(1 + \frac{\mu v^2}{gR}\right) = cos\alpha(\frac{v^2}{gR} - \mu)$$

Or

$\frac{sin\alpha}{cos\alpha} = \frac{(\frac{v^2}{gR} - \mu)}{(1 + \frac{\mu v^2}{gR})} = tan\alpha$

Since μ is a very small value i.e 0.15,

$\left(1 + \frac{\mu v^2}{gR}\right)$ $tends\ to\ zero.$

Hence $tan\alpha\ \frac{v^2}{gR} - \mu$

But $tan\alpha = e$

So $e = \frac{v^2}{gR} - \mu$

$e + \mu = \frac{v^2}{gR} = (\frac{0.28V}{9.8R})^2 = \frac{v^2}{127R}$

Note: if $\mu = 0$ and the forces acting on the vehicle are in equilibrium, then the situation occurs where the centrifugal force is entirely counteracted by the superelevation.

Example

Calculate the superelevation required for a road 7.5m wide in a curve of 240 m radius for a permissible speed of 80 km/h. Asumme the coefficient of internal friction as 0.15. also calculate the equilibrium superelevation for the condition when the pressure on inner and outer wheels will be equal.

Solution

$e + \mu = \frac{v^2}{gR} = (\frac{0.28V}{9.8R})^2 = \frac{v^2}{127R}$

$\mu = 0.15, v = 80\frac{km}{h}, R = 240m$

Therefore

$e + 0.15 = \frac{80^2}{127x240}$

E = 0.21 - 0.15 = 0.06

Therefore, superelevation = 0.06 x 7.5 x 100

45 cm in terms of outer edge over inner

For equilibrium superelevation when the pressure on the inner and outer wheels is to be equal, then $\mu = 0$.

e = $\frac{v^2}{127R} = \frac{80^2}{127x240} = 0.21$

superelevation = 0.21 x 7.5 x 100 = 157.5 cm

in term of rise of outer edge over inner.

Note: if the calculated e is negative, it means that e is not required; i.e $\mu > \frac{v^2}{gR}$

Advantages of providing superelevation

1. It allows for design speed to be maintained on a curve as on a straight portion
2. It helps in keeping the vehicles to their correct side.
3. It lessens the danger of skidding at bends
4. It keeps the pressure on wheels as equally distributed thereby resulting in less wear and tear of wheel tyres and springs.
5. Helps to keep parking lanes generally level
6. To keep the difference in slope between the roadway and any streets or driveways that intersect it within reasonable bounds.
7. It helps to prevent slow moving vehicles from sliding to the inside of the curve.

Superelevation should not be so excessive as to cause a stationary vehicle to slide down the cross slope, regardless of the nature and condition of the road surface. Superelevation rate shall not be less than the rate of crown slope, i.e camber or cross fall.

Superelevation slopes on curves shall extend the full width of shoulders, except that the shoulder slope on the low side shall not be less than the minimum shoulder slope used on tangents.

Example

Calculate the allowable speed on a horizontal curve of radius 200 m. given the following data.

1. Coefficient of lateral friction is 0.15
2. Maximum superelevation of 1 in 15 is not to be exceeded

$e + \mu = \frac{v^2}{127R}$

$R = \frac{v^2}{127(e+\mu)}$;

e = $\frac{1}{15}$

$v^2 = 127R(e + \mu)$

$v^2 = 127x\ 200(0.067 + 0.15)$

$v^2 = 5503.33$

$v = 74.18\ km/h$

Pavement widening

Pavement on curves sometimes are widened to make operating conditions on curves comparable to those on tangents. Pavements widening is needed on certain open highways because the vehicle or truck occupies greater width, since the rear wheels generally track inside front wheels in rounding curves and the drivers have some difficulty in steering their vehicles to hold to the centre of the lane.

Widening should be attained gradually on the approaches to the curve to ensure a reasonablr smooth alignment on the edge of pavement and to fit the paths of vehicles entering or leaving the curve. The following are the principal points of concern in design, they apply to both ends of the highway curves.

On simple curves, widening should be applied on the inside edge of pavement only. The final marked centerline and desirably any central longitudinal point should be placed midway between the edges of the widened pavements.