SECTION 7
DESIGN OF BUILDING MEMBERS

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Steel members in building structures can be part of the floor framing system to carry gravity loads, the vertical framing system, the lateral framing system to provide lateral stability to the building and resist lateral loads, or two or more of these systems. Floor members are normally called **joists, purlins, beams,** or **girders.** Roof members are also known as **rafters.**

Purlins, which support floors, roofs, and decks, are relatively close in spacing. Beams are floor members supporting the floor deck. Girders are steel members spanning between columns and usually supporting other beams. Transfer girders are members that support columns and transfer loads to other columns. The primary stresses in joists, purlins, beams, and girders are due to flexural moments and shear forces.

Vertical members supporting floors in buildings are designated **columns.** The most common steel shapes used for columns are wide-flange sections, pipes, and tubes. Columns are subject to axial compression and also often to bending moments. Slenderness in columns is a concern that must be addressed in the design.

Lateral framing systems may consist of the floor girders and columns that support the gravity floor loads but with rigid connections. These enable the flexural members to serve the dual function of supporting floor loads and resisting lateral loads. Columns, in this case, are subject to combined axial loads and moments. The lateral framing system also can consist of vertical diagonal braces or shear walls whose primary function is to resist lateral loads. Mixed bracing systems and rigid steel frames are also common in tall buildings.

Most steel floor framing members are considered simply supported. Most steel columns supporting floor loads only are considered as pinned at both ends. Other continuous members, such as those in rigid frames, must be analyzed as plane or space frames to determine the members' forces and moments.

Other main building components are steel trusses used for roofs or floors to span greater lengths between columns or other supports, built-up plate girders and stub girders for long spans or heavy loads, and open-web steel joists. See also Sec. 8.

This section addresses the design of these elements, which are common to most steel buildings, based on allowable stress design (ASD) and load and resistance factor design (LRFD). Design criteria for these methods are summarized in Sec. 6.

7.1 TENSION MEMBERS

Members subject to tension loads only include hangers, diagonal braces, truss members, and columns that are part of the lateral bracing system with significant uplift loads.

The AISC "LRFD Specification for Structural Steel Buildings." American Institute of Steel Construction (AISC) gives the nominal strength P_n (kips) of a cross section subject to tension only as the smaller of the capacity of yielding in the gross section,

$$P_n = F_y A_g \tag{7.1}$$

or the capacity at fracture in the net section,

$$P_n = F_u A_e \tag{7.2}$$

The factored load may not exceed either of the following:

$$P_u = \phi F_y A_g \qquad \phi = 0.9 \tag{7.3}$$

$$P_u = \phi F_u A_e \qquad \phi = 0.75 \tag{7.4}$$

where F_y and F_u are, respectively, the yield strength and the tensile strength (ksi) of the member. A_g is the gross area (in^2) of the member, and A_e is the effective cross-sectional area at the connection.

The effective area A_e is given by

$$A_e = UA \tag{7.5}$$

where A = area as defined below
U = reduction coefficient
= $1 - (\bar{x}/L) \leq 0.9$ or as defined below
$\bar{x}$ = connection eccentricity, in
L = length of connection in the direction of loading, in

(a) When the tension load is transmitted only by bolts or rivets:

$$A = A_n$$

$$= \text{net area of the member, in}^2$$

(b) When the tension load is transmitted only by longitudinal welds to other than a plate member or by longitudinal welds in combination with transverse welds:

$$A = A_g$$

$$= \text{gross area of member, in}^2$$

(c) When the tension load is transmitted only by transverse welds:

$$A = \text{area of directly connected elements, in}^2$$

$$U = 1.0$$

(d) When the tension load is transmitted to a plate by longitudinal welds along both edges at the end of the plate for $l \geq w$:

$$A = \text{area of plate, in}^2$$

where U = 1.00 when $l > 2w$
= 0.87 when $2w > l \geq 1.5w$
= 0.75 when $1.5w > l \geq w$
l = weld length, in $> w$
w = plate width (distance between welds), in

7.2 COMPARATIVE DESIGNS OF DOUBLE-ANGLE HANGER

A composite floor framing system is to be designed for sky boxes of a sports arena structure. The sky boxes are located about 15 ft below the bottom chord of the roof trusses. The sky-box framing is supported by an exterior column at the exterior edge of the floor and by steel hangers 5 ft from the inside edge of the floor. The hangers are connected to either the bottom chord of the trusses or to the steel beams spanning between trusses at roof level. The reactions due to service dead and live loads at the hanger locations are $P_{DL} = 55$ kips and $P_{LL} = 45$ kips. Hangers supporting floors and balconies should be designed for additional impact factors representing 33% of the live loads.

7.2.1 LRFD for Double-Angle Hanger

The factored axial tension load is the larger of

$$P_{UT} = 55 \times 1.2 + 45 \times 1.6 \times 1.33 = 162 \text{ kips (governs)}$$

$$P_{UT} = 55 \times 1.4 = 77 \text{ kips}$$

Double angles of A36 steel with one row of three bolts at 3 in spacing will be used ($F_y = 36$ ksi and $F_u = 58$ ksi). The required area of the section is determined as follows: From Eq. (7.3), with $P_U = 162$ kips,

$$A_g = 162/(0.9 \times 36) = 5.00 \text{ in}^2$$

From Eq. (7.4),

$$A_e = 162/(0.75 \times 58) = 3.72 \text{ in}^2$$

Try two angles, $5 \times 3 \times 3/8$ in, with $A_g = 5.72$ in^2. For 1-in-diameter A325 bolts with hole size $1\frac{1}{16}$ in, the net area of the angles is

$$A_n = 5.72 - 2 \times 3/8 \times 17/16 = 4.92 \text{ in}^2$$

and

$$U = 1 - (\bar{x}/L) = 1 - (0/9) = 1.0 > 0.9$$

Therefore, $U = 0.9$
The effective area is

$$A_e = UA_n = 0.90 \times 4.92 = 4.43 \text{ in}^2 > 3.72 \text{ in}^2\text{—OK}$$

7.2.2 ASD for Double-Angle Hanger

The dead load on the hanger is 55 kips, and the live load plus impact is $45 \times 1.33 = 60$ kips (Art. 7.2.1). The total axial tension then is $55 + 60 = 115$ kips. With the allowable tensile stress on the gross area of the hanger $F_1 = 0.6F_y = 0.6 \times 36 = 21.6$ ksi, the gross area A_g required for the hanger is

$$A_g = 115/21.6 = 5.32 \text{ in}^2$$

With the allowable tensile stress on the effective net area $F_t = 0.5F_u = 0.5 \times 58 = 29$ ksi,

$$A_e = 115/29 = 3.97 \text{ in}^2$$

Two angles 5 × 3 × ⅜ in provide $A_g = 5.72 \text{ in}^2 > 5.32 \text{ in}^2$—OK. For 1-in-diameter bolts in holes 1¹⁄₁₆ in in diameter, the net area of the angles is

$$A_n = 5.72 - 2 \times 3/8 \times 17/16 = 4.92 \text{ in}^2$$

and the effective net area is

$$A_e = UA_n = 0.85 \times 4.92 = 4.18 \text{ in}^2 > 3.97 \text{ in}^2\text{—OK}$$

7.3 EXAMPLE—LRFD FOR WIDE-FLANGE TRUSS MEMBERS

One-way, long-span trusses are to be used to frame the roof of a sports facility. The truss span is 300 ft. All members are wide-flange sections. (See Fig. 7.1 for the typical detail of the bottom-chord splice of the truss).

Connections of the truss diagonals and verticals to the bottom chord are bolted. Slip-critical, the connections serve also as splices, with 1⅛-in-diameter A325 bolts, in oversized holes to facilitate truss assembly in the field. The holes are 1⁷⁄₁₆ in in diameter. The bolts are placed in two rows in each flange. The number of bolts per row is more than two. The web of each member is also spliced with a plate with two rows of 1⅛-in-diameter A325 bolts.

The structural engineer analyzes the trusses as pin-ended members. Therefore, all members are considered to be subject to axial forces only. Members of longspan trusses with significant deflections and large, bolted, slip-critical connections, however, may have significant bending moments. (See Art. 7.15 for an example of a design for combined axial load and bending moments.)

The factored axial tension in the bottom chord at midspan due to combined dead, live, theatrical, and hanger loads supporting sky boxes is $P_u = 2280$ kips.

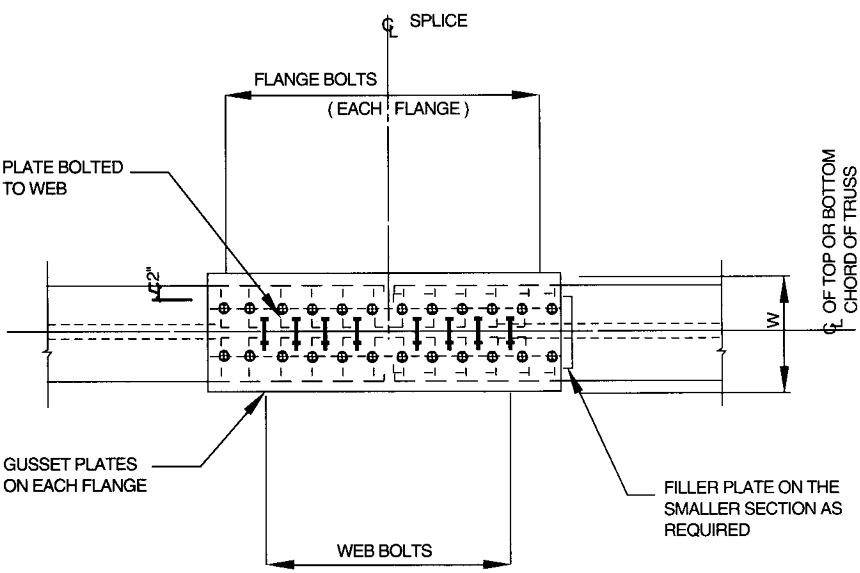

FIGURE 7.1 Detail of a splice in the bottom chord of a truss.

With a wide-flange section of grade 50 steel (F_y = 50 ksi and F_u = 65 ksi), the required minimum gross area, from Eq. (7.3), is

$$A_g = P_u/\phi F_y = 2280/(0.9 \times 50) = 50.67 \text{ in}^2$$

Try a W14 × 176 section with A_g = 51.8 in², flange thickness t_f = 1.31 in, and web thickness t_w = 0.83 in. The net area is

$$\begin{aligned} A_n &= 51.8 - (2 \times 1.31 \times 1.4375 \times 2 + 2 \times 0.83 \times 1.4375) \\ &= 41.88 \text{ in}^2 \end{aligned}$$

Since all parts of the wide-flange section are connected at the splice connection, $U = 1$ for determination of the effective area from Eq. (7.5). Thus $A_e = A_n = 41.88$ in². From Eq. (7.4), the design strength is

$$\phi P_n = 0.75 \times 65 \times 41.88 = 2042 \text{ kips} < 2280 \text{ kips—NG}$$

Try a W14 × 193 with A_g = 56.8 in², t_f = 1.44 in, and t_w = 0.89 in. The net area is

$$\begin{aligned} A_n &= 56.8 - (2 \times 1.44 \times 1.4375 \times 2 + 2 \times 0.89 \times 1.4375) \\ &= 45.96 \text{ in}^2 \end{aligned}$$

From Eq. (7.4), the design strength is

$$\phi P_n = 0.75 \times 65 \times 45.96 = 2241 \text{ kips} < 2280 \text{ ksi—NG}$$

Use the next size, W14 × 211.

7.4 COMPRESSION MEMBERS

Steel members in buildings subject to compressive axial loads include columns, truss members, struts, and diagonal braces. Slenderness is a major factor in design of compression members. The slenderness ratio L/r is preferably limited to 200. Most suitable steel shapes are pipes, tubes, or wide-flange sections, as designated for columns in the AISC "Steel Construction Manual." Double angles, however, are commonly used for diagonal braces and truss members. Double angles can be easily connected to other members with gusset plates and bolts or welds.

The AISC "LRFD Specification for Structural Steel Buildings," American Institute of Steel Construction, gives the nominal strength P_n (kips) of a steel section in compression as

$$P_n = A_g F_{cr} \tag{7.6}$$

The factored load P_u (kips) may not exceed

$$P_u = \phi P_n \qquad \phi = 0.85 \tag{7.7}$$

The critical compressive stress F_{cr} (kips) is a function of material strength and slenderness. For determination of this stress, a column slenderness parameter λ_c is defined as

$$\lambda_c = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} = \frac{KL}{r}\sqrt{\frac{F_y}{286{,}220}} \tag{7.8}$$

where A_g = gross area of the member, in²
K = effective length factor (Art. 6.16.2)

L = unbraced length of member, in
F_y = yield strength of steel, ksi
E = modulus of elasticity of steel material, ksi
r = radius of gyration corresponding to plane of buckling, in

When $\lambda_c \leq 1.5$, the critical stress is given by

$$F_{cr} = (0.658^{\lambda_c^2})F_y \tag{7.9}$$

When $\lambda_c > 1.5$,

$$F_{cr} = \left(\frac{0.877}{\lambda_c^2}\right) F_y \tag{7.10}$$

7.5 EXAMPLE—LRFD FOR STEEL PIPE IN AXIAL COMPRESSION

Pipe sections of A36 steel are to be used to support framing for the flat roof of a one-story factory building. The roof height is 18 ft from the tops of the steel roof beams to the finish of the floor. The steel roof beams are 16 in deep, and the bases of the steel-pipe columns are 1.5 ft below the finished floor. A square joint is provided in the slab at the steel column. Therefore, the concrete slab does not provide lateral bracing. The effective height of the column, from the base of the column to the center line of the steel roof beam, is

$$h = 18 + 1.5 - \frac{16}{2 \times 12} = 18.83 \text{ ft}$$

The dead load on the column is 30 kips. The live load due to snow at the roof is 36 kips. The factored axial load is the larger of the following:

$$P_u = 30 \times 1.4 = 42 \text{ kips}$$

$$P_u = 30 \times 1.2 + 36 \times 1.6 = 93.6 \text{ kips (governs)}$$

With the factored load known, the required pipe size may be obtained from a table in the AISC "Manual of Steel Construction—LRFD." For $KL = 19$ ft, a standard 6-in pipe (weight 18.97 lb per linear ft) offers the least weight for a pipe with a compression-load capacity of at least 93.6 kips. For verification of this selection, the following computations for the column capacity were made based on a radius of gyration $r = 2.25$ in. From Eq. (7.8),

$$\lambda_c = \frac{18.83 \times 12}{2.25} \sqrt{\frac{36}{286{,}220}} = 1.126 < 1.5$$

and $\lambda_c^2 = 1.269$. For $\lambda_c < 1.5$, Eq. (7.9) yields the critical stress

$$F_{cr} = 0.658^{1.269} \times 36 = 21.17 \text{ ksi}$$

The design strength of the 6-in pipe, then, from Eqs. (7.6) and (7.7), is

$$\phi P_n = 0.85 \times 5.58 \times 21.17 = 100.4 \text{ kips} > 93.6 \text{ kips—OK}$$

7.6 COMPARATIVE DESIGNS OF WIDE-FLANGE SECTION WITH AXIAL COMPRESSION

A wide-flange section is to be used for columns in a five-story steel building. A typical interior column in the lowest story will be designed to support gravity loads. (In this example, no eccentricity will be assumed for the load.) The effective height of the column is 18 ft. The axial loads on the column from the column above and from the steel girders supporting the second level are dead load 420 kips and live load (reduced according to the applicable building code) 120 kips.

7.6.1 LRFD for W Section with Axial Compression

The factored axial load is the larger of the following:

$$P_u = 420 \times 1.4 = 588 \text{ kips}$$

$$P_u = 420 \times 1.2 + 120 \times 1.6 = 696 \text{ kips (governs)}$$

To select the most economical section and material, assume that grade 36 steel costs \$0.24 per pound and grade 50 steel costs \$0.26 per pound at the mill. These costs do not include the cost of fabrication, shipping, or erection, which will be considered the same for both grades.

Use of the column design tables of the AISC "Manual of Steel Construction—LRFD" presents the following options:

For the column of grade 36 steel, select a W14 × 99, with a design strength $\phi P_n = 745$ kips.

$$\text{Cost} = 99 \times 18 \times 0.24 = \$428$$

For the column of grade 50 steel, select a W12 × 87, with a design strength $\phi P_n = 758$ kips.

$$\text{Cost} = 87 \times 18 \times 0.26 = \$407$$

Therefore, the W12 × 87 of grade 50 steel is the most economical wide-flange section.

7.6.2 ASD for W Section with Axial Compression

The dead- plus live-load axial compression totals 420 + 120 = 540 kips (Art. 7.6.1).

Column design tables in the AISC "Steel Construction Manual—ASD" facilitate selection of wide-flange sections for various loads for columns of grades 36 and 50 steels.

For the column of grade 36 steel, with the slenderness ratio $KL = 18$ ft, the manual tables indicate that the least-weight section with a capacity exceeding 540 kips is a W14 × 109. It has an axial load capacity of 564 kips. Estimated cost of the W14 × 109 is \$0.24 × 109 × 18 = \$471.

LRFD requires a W14 × 99 of grade 36 steel, with an estimated cost of \$428. Thus the cost savings by use of LRFD is 100(471 − 428)/428 = 9.1%.

For the column of grade 50 steel, with $KL = 18$ ft, the manual tables indicate that the least-weight section with a capacity exceeding 540 kips is a W14 × 90. It has an axial compression capacity of 609 kips. Estimated cost of the W14 × 90 is \$0.26 × 90 × 18 = \$421. Thus the grade 50 column costs less than the grade 36 column.

LRFD requires a W12 × 87 of grade 50 steel, with an estimated cost of \$407. The cost savings by use of LRFD is 100(421 − 407)421 = 3.33%.

This example indicates that when slenderness is significant in design of compression members, the savings with LRFD are not as large for slender members as for stiffer members, such as short columns or columns with a large radius of gyration about the x and y axes.

7.7 EXAMPLE—LRFD FOR DOUBLE ANGLES WITH AXIAL COMPRESSION

Double angles are the preferred steel shape for a diagonal in the vertical bracing part of the lateral framing system in a multistory building (Fig. 7.2). Lateral load on the diagonal in this example is due to wind only and equals 65 kips. The diagonals also support the steel beam at midspan. As a result, the compressive force on each brace due to dead loads is 15 kips, and that due to live loads is 10 kips. The maximum combined factored load is $P_u = 1.2 \times 15 + 1.3 \times 65 + 0.5 \times 10 = 107.5$ kips.

The length of the brace is 19.85 ft, neglecting the size of the joint. Grade 36 steel is selected because slenderness is a major factor in determining the nominal capacity of the section. Selection of the size of double angles is based on trial and error, which can be assisted by load tables in the AISC "Manual of Steel Construction—LRFD" for columns of various shapes and sizes. For the purpose of illustration of the step-by-step design, double angles 6 × 4 × ⅝ in with ⅜-in spacing between the angles are chosen. Section properties are as follows: gross area $A_g = 11.7$ in^2 and the radii of gyration are $r_x = 1.90$ in and $r_y = 1.67$ in.

First, the slenderness effect must be evaluated to determine the corresponding critical compressive stresses. The effect of the distance between the spacer plates connecting the two angles is a design consideration in LRFD. Assuming that the connectors are fully tightened bolts, the system slenderness is calculated as follows:

The AISC "LRFD Specification for Structural Steel Buildings" defines the following modified column slenderness for a built-up member:

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\,\frac{\alpha^2}{1+\alpha^2}\left(\frac{a}{r_{ib}}\right)^2} \tag{7.11}$$

where: $\left(\frac{KL}{r}\right)_o$ = column slenderness of built-up member acting as a unit

α = separation ratio = $h/2r_{ib}$

h = distance between centroids of individual components perpendicular to member axis of buckling

a = distance between connectors

r_{ib} = radius of gyration of individual angle relative to its centroidal axis parallel to member axis of buckling

In this case, $h = 1.03 + 0.375 + 1.03 = 2.44$ in and $\alpha = 2.44/(2 \times 1.13) = 1.08$. Assume maximum spacing between connectors is $a = 80$ in. With $K = 1$, substitution in Eq. 7.11 yields

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{19.85 \times 12}{1.67}\right)^2 + 0.82\,\frac{1.08^2}{1 + 1.08^2}\left(\frac{80}{1.13}\right)^2} = 150$$

From Eq. 7.8, for determination of the critical stress F_{cr},

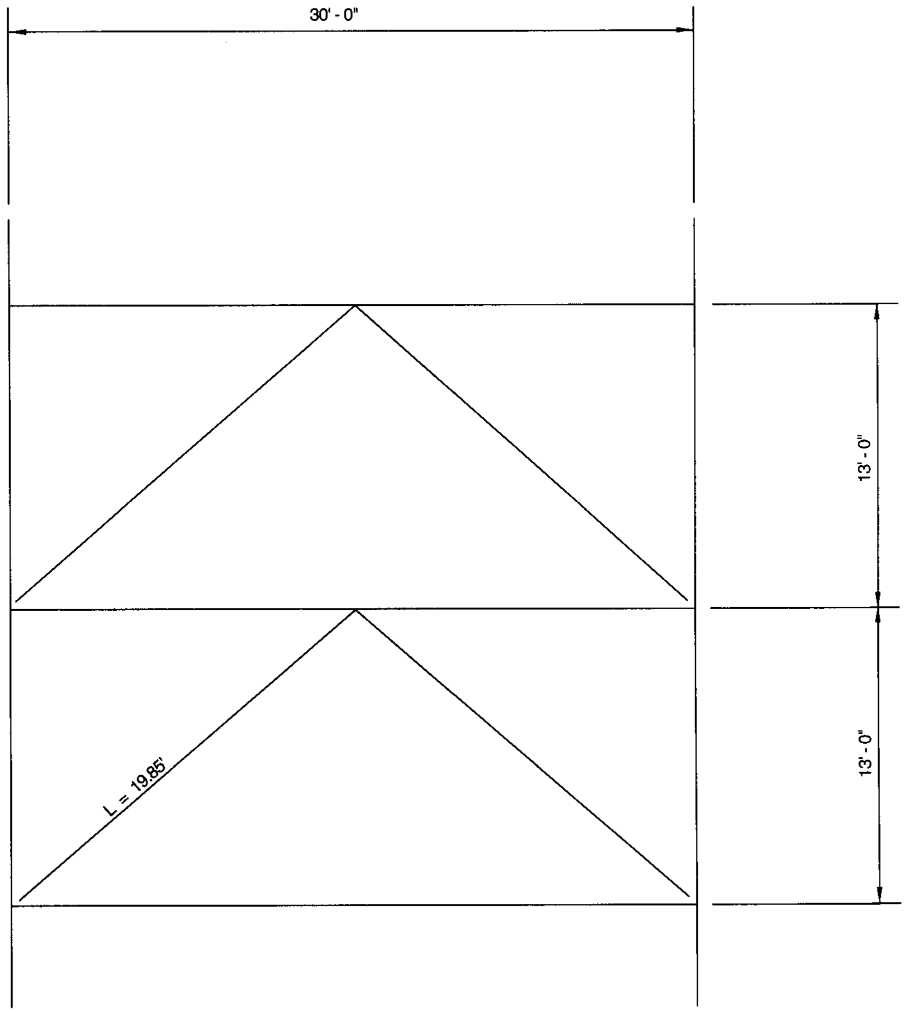

FIGURE 7.2 Inverted V-braces in a lateral bracing bent.

$$\lambda_c = 150\sqrt{\frac{36}{286{,}220}} = 1.68 > 1.5$$

The critical stress, from Eq. (7.10), then is

$$F_{cr} = \left(\frac{0.877}{1.68^2}\right)36 = 11.19 \text{ ksi}$$

From Eqs. (7.6) and (7.7), the design strength is

$$\phi P_n = 0.85 \times 11.7 \times 11.19 = 111.3 \text{ kips} > 107.5 \text{ kips—OK}$$

7.8 STEEL BEAMS

According to the AISC "LRFD Specification for Structural Steel Buildings," the nominal capacity M_p (in-kips) of a steel section in flexure is equal to the plastic moment:

$$M_p = ZF_y \tag{7.12}$$

where Z is the plastic section modulus (in^3), and F_y is the steel yield strength (ksi). But this applies only when local or lateral torsional buckling of the compression flange is not a governing criterion. The nominal capacity M_p is reduced when the compression flange is not braced laterally for a length that exceeds the limiting unbraced length for full plastic bending capacity L_p. Also, the nominal moment capacity is less than M_p, when the ratio of the compression-element width to its thickness exceeds limiting slenderness parameters for compact sections. The same is true for the effect of the ratio of web depth to thickness. (See Arts. 6.17.1 and 6.17.2.)

In addition to strength requirements for design of beams, serviceability is important. Deflection limitations defined by local codes or standards of practice must be maintained in selecting member sizes. Dynamic properties of the beams are also important design parameters in determining the vibration behavior of floor systems for various uses.

The shear forces in the web of wide-flange sections should be calculated, especially if large concentrated loads occur near the supports. The AISC specification requires that the factored shear V_v (kips) not exceed

$$V_u = \phi_u V_n \qquad \phi_v = 0.90 \tag{7.13}$$

where ϕ_v is a capacity reduction factor and V_n is the nominal shear strength (kips). For $h/t_w \leq 187\sqrt{k_v/F_{yw}}$,

$$V_n = 0.6F_{yw}A_w \tag{7.14}$$

where h = clear distance between flanges (less the fillet or corner radius for rolled shapes), in
k_v = web-plate buckling coefficient (Art. 6.14.1)
t_w = web thickness, in
F_{yw} = yield strength of the web, ksi
A_w = web area, in^2

For $187\sqrt{k_v/F_{yw}} < h/t_w \leq 234\ \sqrt{k_v/F_{yw}}$,

$$V_n = 0.6F_{yw}A_w \frac{187\sqrt{k_v/F_{yw}}}{h/t_w} \tag{7.15}$$

For $h/t_w > 234\ \sqrt{k_v/F_{yw}}$,

$$V_n = A_w \frac{26{,}400k_v}{(h/t_w)^2} \tag{7.16}$$

$$\begin{aligned} k_v &= 5 + 5/(a + h)^2 \\ &= 5 \text{ when } a/h > 3 \text{ or } a/h > [260/(h/t)]^2 \\ &= 5 \text{ if no stiffeners are used} \end{aligned} \tag{7.17}$$

where a = distance between transverse stiffeners

7.9 COMPARATIVE DESIGNS OF SIMPLE-SPAN FLOORBEAM

Floor framing for an office building is to consist of open-web steel joists with a standard corrugated metal deck and 3-in-thick normal-weight concrete fill. The joists are to be spaced 3 ft center to center. Steel beams spanning 30 ft between columns support the joists. A bay across the building floor is shown in Fig. 7.3.

Floorbeam *AB* in Fig. 7.3 will be designed for this example. The loads are listed in Table 7.1. The live load is reduced in Table 7.1, as permitted by the *Uniform Building Code.* The reduction factor *R* is given by the smaller of

$$R = 0.0008(A - 150) \tag{7.18}$$

$$R = 0.231(1 + D/L) \tag{7.19}$$

$$R = 0.4 \text{ for beams} \tag{7.20}$$

where D = dead load
L = live load
A = area supported = $30(40 + 25)/2 = 975$ ft^2

From Eq. (7.18), $R = 0.0008(975 - 150) = 0.66$.
From Eq. (7.19), $R = 0.231(1 + 73/50) = 0.568$.
From Eq. (7.20), $R = 0.4$ (governs), and the reduced live load is $50(10.4) = 30$ lb per ft^2, as shown in Table 7.1.

7.9.1 LRFD for Simple-Span Floorbeam

If the beam's self-weight is assumed to be 45 lb/ft, the factored uniform load is the larger of the following:

$$W_u = 1.4[73(40 + 25)/2 + 45] = 3384.5 \text{ lb per ft}$$

$$W_u = 1.2[73(40 + 25)/2 + 45] + 1.6 \times 30(40 + 25)/2$$
$$= 4461 \text{ lb per ft (governs)}$$

The factored moment then is

$$M_u = 4.461(30)^2/8 = 501.9 \text{ kip-ft}$$

To select for beam *AB* a wide-flange section with $F_y = 50$ ksi, the top flange being braced by joists, the required plastic modulus Z_x is determined as follows:
The factored moment M_u may not exceed the design strength of ϕM_r, and

$$\phi M_r = \phi Z_x F_y \tag{7.21}$$

Therefore, from Eq. (7.21),

$$Z_x = \frac{501.9 \times 12}{0.9 \times 50} = 133.8 \text{ in}^3$$

A wide-flange section W24 × 55 with $Z = 134$ in^3 is adequate.

Next, criteria are used to determine if deflections are acceptable. For the live-load deflection, the span L is 30 ft, the moment of inertia of the W24 × 55 is $l = 1350$ in^4, and

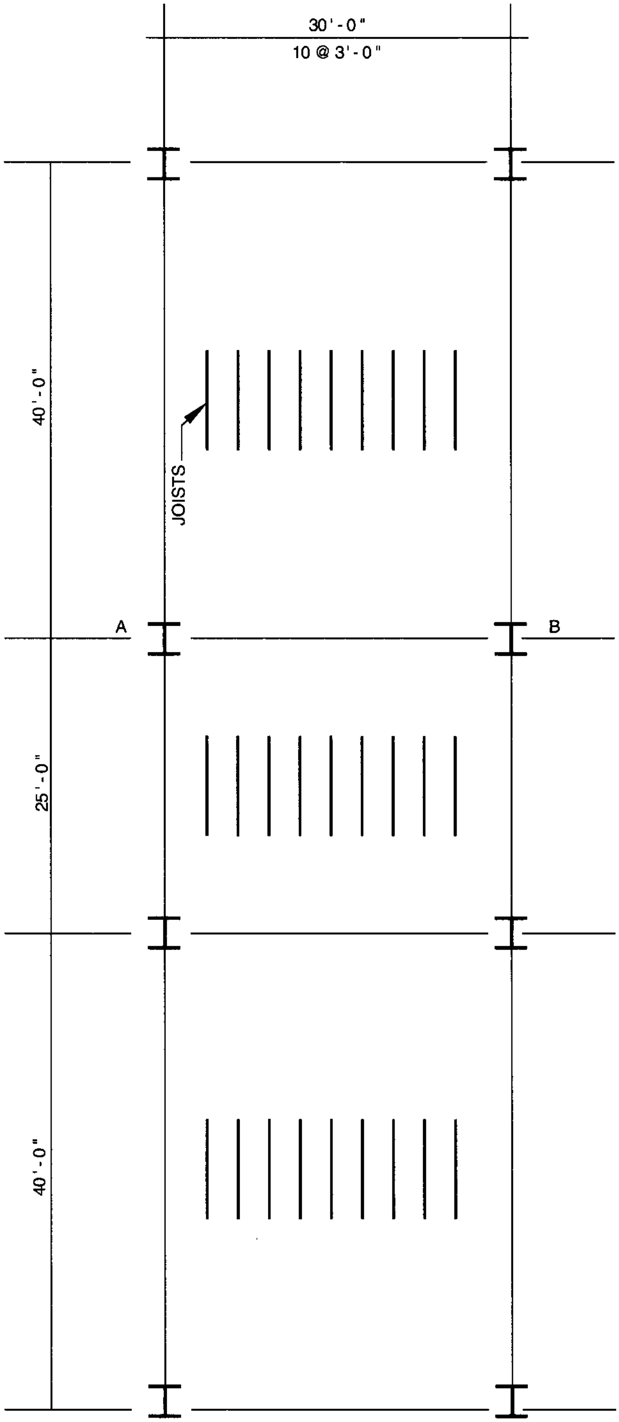

FIGURE 7.3 Part of the floor framing for an office building.

TABLE 7.1 Loads on Floorbeam *AB* in Fig. 7.3

Dead loads, lb per ft^2	
Floor deck	45
Ceiling and mechanical ductwork	5
Open-web joists	3
Partitions	20
Total dead load (exclusive of beam weight)	73
Live loads, lb per ft^2	
Full live load	50
Reduced live load: 50(1 − 0.4)	30

the modulus of elasticity $E = 29{,}000$ ksi. The live load is $W_L = 30(40 + 25)/2 = 975$ lb per ft. Hence the live-load deflection is

$$\Delta_L = \frac{5W_LL^4}{394EI} = \frac{5 \times 0.975 \times 30^4 \times 12^3}{384 \times 29{,}000 \times 1{,}350} = 0.454 \text{ in}$$

This value is less than $L/360 = 30 \times {}^{12}\!/_{360} = 1$ in, as specified in the *Uniform Building Code* (*UBC*). The *UBC* requires that deflections due to live load plus a factor K times deadload not exceed $L/240$. The K value, however, is specified as zero for steel. [The intent of this requirement is to include the long-term effect (creep) due to dead loads in the deflection criteria.] Hence the live-load deflection satisfies this criterion.

The immediate deflection due to the weight of the concrete and the floor framing is also commonly determined. If excessive deflections due to such dead loads are found, it is recommended that steel members be cambered to produce level floors and to avoid excessive concrete thickness during finishing the wet concrete.

In this example, the load due to the weight of the floor system is from Table 7.1 with the weight of the beam added,

$$W_{wt} = (45 + 3)(40 + 25)/2 + 55 = 1615 \text{ lb per ft}$$

The deflection due to this load is

$$\Delta_{wt} = \frac{5 \times 1.615 \times 30^4 \times 12}{384 \times 29{,}000 \times 1{,}350} = 0.752 \text{ in}$$

Therefore, cambering the beam ¾ in at midspan is recommended.

For review of the shear capacity of the section, the depth/thickness ratio of the web is

$$h/t_w = 54.6 < (187\sqrt{5/50} = 59.13)$$

From Eq. (7.14), the design shear strength is

$$\phi V_n = 0.9 \times 0.6 \times 50 \times 23.57 \times 0.395 = 251 \text{ kips}$$

The factored shear force near the support is

$$V_u = 4.461 \times 30/2 = 66.92 \text{ kips} < 251 \text{ kips—OK}$$

As illustrated in this example, it usually is not necessary to review the design of each simple beam with uniform load for shear capacity.

7.9.2 ASD for Simple-Span Floorbeam

The maximum moment due to the dead and live loads provided for Art. 7.9.1 is calculated as follows.

The total service load, after allowing a reduction in live load for size of area supported, is $73 + 30 = 103$ lb per ft^2. Assume that the beam weighs 60 lb per ft. Then, the total uniform load on the beam is

$$W_t = 103 \times 0.5(40 + 25) + 60 = 3408 \text{ lb per ft} = 3.408 \text{ kips per ft}$$

For this load, the maximum moment is

$$M = 3.408 \times 30^2/8 = 383.4 \text{ kip-ft}$$

For an allowable stress $F_b = 0.66F_y = 0.66 \times 50 = 33$ ksi, the required section modulus for the floorbeam is

$$S_r = M/F_b = 383.4 \times 12/33 = 139.4 \text{ in}^3$$

The least-weight wide-flange section with S exceeding 139.4 is a W21 $\times$ 68 or a W24 $\times$ 68. (If depth is not important, choose the latter because it will deflect less.)

LRFD requires a W24 $\times$ 55. The weight saving with LRFD is $100(68 - 55)/68 = 19.1\%$.

The percentage savings in weight with LRFD differs significantly from that in this example for a different ratio of live load to dead load. When live loads are relatively large, such as 100 psf for occupancy load in public areas, the savings in steel tonnage with LRFD is not as large as this example indicates.

Deflection calculation for ASD of the floorbeam is similar to that performed in Art. 7.9.1.

For review of shear stresses, the depth/thickness ratio of the web of the W24 $\times$ 68 is $h/t_w = 21/0.415 = 50.6$. Since this is less than $380/\sqrt{F_y} = 380/\sqrt{50} = 53.7$, the allowable shear stress is $F_v = 0.4 \times 50 = 20$ ksi. The vertical shear at the support is $V = 3.408 \times 30/2 = 51.12$ kips. Hence the shear stress there is

$$f_v = 51.12/23.73 \times 0.415 = 5.19 \text{ ksi} < 20 \text{ ksi—OK}$$

7.10 *EXAMPLE—LRFD FOR FLOORBEAM WITH UNBRACED TOP FLANGE*

A beam of grade 50 steel with a span of 20 ft is to support the concentrated load of a stub pipe column at midspan. The factored concentrated load is 55 kips. No floor deck is present on either side of the beam to brace the top flange, and the pipe column is not capable of bracing the top flange laterally. The weight of the beam is assumed to be 50 lb/ft.

The factored moment at midspan is

$$M_u = 55 \times 20/4 + 0.050 \times 20^2/8 = 277.5 \text{ kip-ft}$$

A beam size for a first trial can be selected from a load-factor design table for steel with $F_y = 50$ ksi in the AISC "Steel Construction Manual—LRFD." The table lists several properties of wide-flange shapes, including plastic moment capacities ϕM_p. For example, an examination of the table indicates that the lightest beam with ϕM_p exceeding 277.5 kip-ft is a W18 $\times$ 40 with $\phi M_p = 294$ kip-ft. Whether this beam can be used, however, depends on the resistance of its top flange to buckling. The manual table also lists the limiting laterally unbraced lengths for full plastic bending capacity L_p and inelastic torsional buckling L_r. For the W18 $\times$ 40, $L_p = 4.5$ ft and $L_r = 12.1$ ft (Table 7.2).

TABLE 7.2 Properties of Selected W Shapes for LRFD

Property	W18 × 35 grade 36	W18 × 40 grade 50	W21 × 50 grade 50	W21 × 62 grade 50
ϕM_p, kip-ft	180	294	413	540
L_p, ft	5.1	4.5	4.6	6.3
L_r, ft	14.8	12.1	12.5	16.6
ϕM_r, kip-ft	112	205	283	381
S_x, in^3	57.6	68.4	94.5	127
X_1, ksi	1590	1810	1730	1820
X_2, 1/ksi^2	0.0303	0.0172	0.0226	0.0159
r_y, in	1.22	1.27	1.30	1.77

In this example, then, the 20-ft unbraced beam length exceeds L_r. For this condition, the nominal bending capacity M_n is given by Eq. (6.54): $M_n = M_{cr} \leq C_n M_r$. For a simple beam with a concentrated load, the moment gradient C_b is unity. From the table in the manual for the W18 × 40 (grade 50), design strength $\phi M_r = 205$ kip-ft < 277.5 kip-ft. Therefore, a larger size is necessary.

The next step is to find a section that if its L_r is less than 20 ft, its M_r exceeds 277.5 kip-ft. The manual table indicated that a W21 × 50 has the required properties (Table 7.2). With the aid of Table 7.2, the critical elastic moment capacity ϕM_{cr} can be computed from

$$\phi M_{cr} = 0.90 \frac{C_b S_x X_1 \sqrt{2}}{L_b / r_y} \sqrt{1 + \frac{X_1{}^2 X_2}{2(L_b/r_y)^2}} \tag{7.22}$$

The beam slenderness ratio with respect to the y axis is

$$L_b/r_y = 20 \times 12/1.30 = 184.6$$

Thus the critical elastic moment capacity is

$$\phi M_{cr} = 0.90 \frac{1 \times 94.5 \times 1{,}730\sqrt{2}}{184.6} \sqrt{1 + \frac{1{,}730^2 \times 0.0226}{2(184.6)^2}}$$

$$= 1{,}591 \text{ kip-in} = 132.6 \text{ kip-ft} < 277.5 \text{ kip-ft}$$

The W21 × 50 does not have adequate capacity. Therefore, trials to find the lowest-weight larger size must be continued. This trial-and-error process can be eliminated by using beam-selector charts in the AISC manual. These charts give the beam design moment corresponding to unbraced length for various rolled sections. Thus for $\phi M_r > 277.5$ kip-ft and $L = 20$ ft, the charts indicate that a W21 × 62 of grade 50 steel satisfies the criteria (Table 7.2). As a check, the following calculation is made with the properties of the W21 × 62 given in Table 7.2.

For use in Eq. (7.22), the beam slenderness ratio is

$$L_b/r_y = 20 \times 12/1.77 = 135.6$$

From Eq. (7.22), the critical elastic moment capacity is

$$\phi M_{cr} = 0.9 \frac{1 \times 127 \times 1820\sqrt{2}}{135.6} \sqrt{1 + \frac{1820^2 \times 0.0159}{(135.6)^2}}$$

$$= 3384 \text{ kip-in} = 282 \text{ kip-ft} > 277.5 \text{ kip-ft—OK}$$

7.11 EXAMPLE—LRFD FOR FLOORBEAM WITH OVERHANG

A floorbeam of A36 steel carrying uniform loads is to span 30 ft and cantilever over a girder for 7.5 ft (Fig. 7.4). The beam is to carry a dead load due to the weight of the floor plus assumed weight of beam of 1.5 kips per ft and due to partitions, ceiling, and ductwork of 0.75 kips per ft. The live load is 1.5 kips per ft.

Negative Moment. The cantilever is assumed to carry full live and dead loads, while the back span is subjected to the minimum dead load. This loading produces maximum negative moment and maximum unbraced length of compression (bottom) flange between the support and points of zero moment. The maximum factored load on the cantilever (Fig. 7.4*a*) is

$$W_{uc} = 1.2(1.5 + 0.75) + 1.6 \times 1.5 = 5.1 \text{ kips per ft}$$

The factored load on the backspan from dead load only is

$$W_{ub} = 1.2 \times 1.5 = 1.8 \text{ kips per ft}$$

Hence the maximum factored moment (at the support) is

$$-M_u = 5.1 \times 7.5^2/2 = 143.4 \text{ kip-ft}$$

From the bending moment diagram in Fig. 7.4*b*, the maximum factored moment in the backspan is 137.1 kip-ft, and the distance between the support of the cantilever and the point of inflection in the backspan is 5.3 ft. The compression flange is unbraced over this distance. The beam will be constrained against torsion at the support. Therefore, since the 7.5-ft cantilever has a longer unbraced length and its end will be laterally braced, design of the section should be based on $L_b = 7.5$ ft.

A beam size for a first trial can be selected from a load-factor design table in the AISC "Steel Construction Manual—LRFD." The table indicates that the lightest-weight section with ϕM_p exceeding 143.4 kip-ft and with potential capacity to sustain the large positive moment in the backspan is a W18 × 35. Table 7.2 lists section properties needed for computation of the design strength. The table indicates that the limiting unbraced length L_r for inelastic torsional buckling is 14.8 ft > L_b. The design strength should be computed from Eq. (6.53):

$$\phi M_n = C_b[\phi M_p - (\phi M_p - \phi M_r)(L_b - L_p)/(L_r - L_p)] \tag{7.23}$$

For an unbraced cantilever, the moment gradient C_b is unity. Therefore, the design strength at the support is

$$\phi M_n = 1[180 - (180 - 112)(7.5 - 5.1)/(14.8 - 5.1)]$$
$$= 163.2 \text{ kip-ft} > 143.4 \text{ kip ft—OK}$$

Positive Moment. For maximum positive moment, the cantilever carries minimum load, whereas the backspan carries full load (Fig. 7.4*c*). Dead load is the minimum for the cantilever:

$$W_{uc} = 1.2 \times 1.5 = 1.8 \text{ kips per ft}$$

Maximum factored load on the backspan is

$$W_{ub} = 1.2(1.5 + 0.75) + 1.6 \times 1.5 = 5.1 \text{ kips per ft}$$

Corresponding factored moments are (Fig. 7.4*d*)

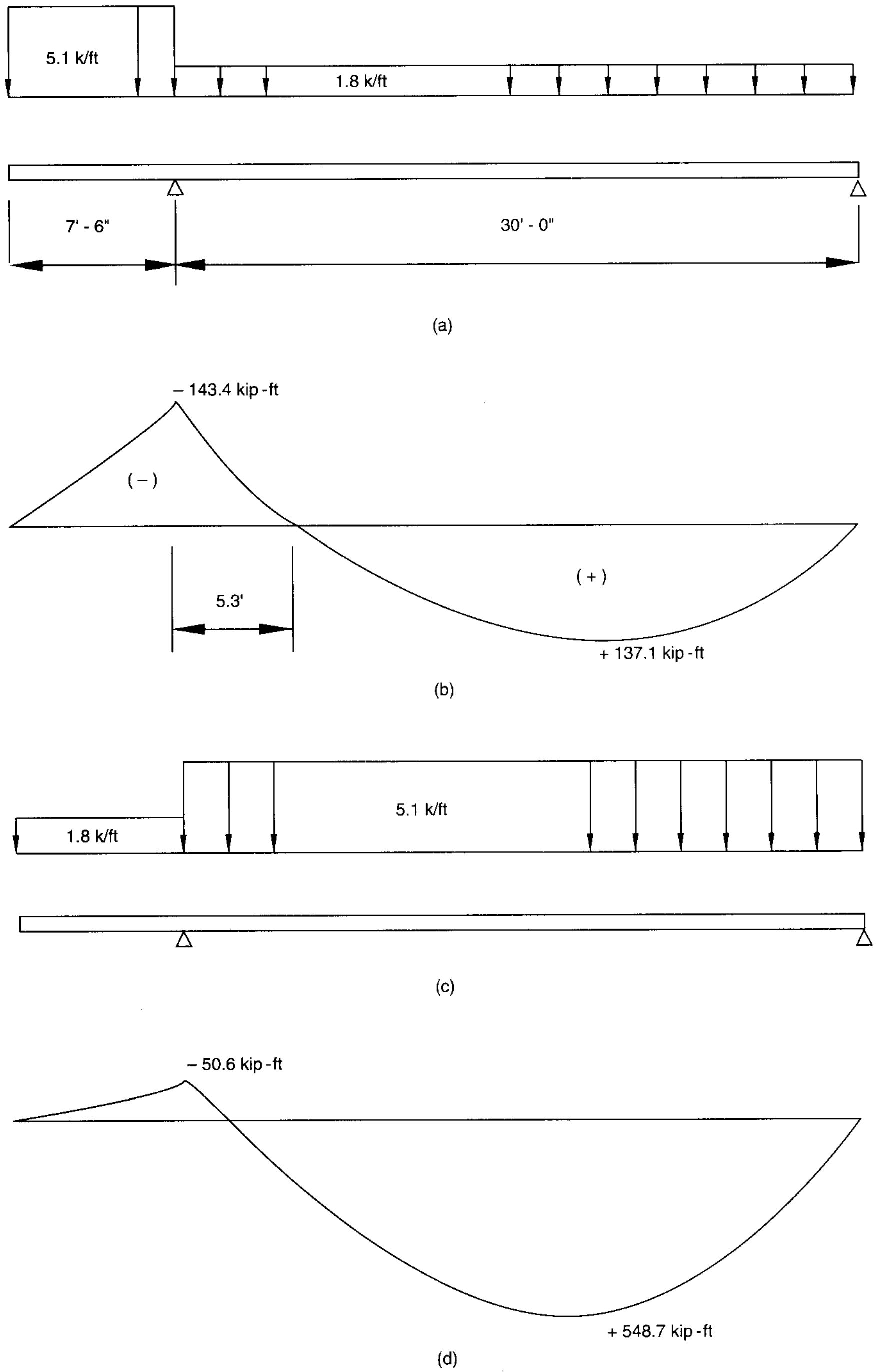

FIGURE 7.4 Loads and moments for a floorbeam with an overhang. (*a*) Placement of factorcd loads for maximum negative moment. (*b*) Factored moments for the loading in (*a*). (*c*) Placement of factored loads for maximum positive moment. (*d*) Factored moments for the loading in (*c*).

$$-M_u = 1.8 \times 7.52^2/2 = 50.6 \text{ kip-ft}$$

$$+M_u = 5.1 \times 30^2/8 - 50.6/2 + \frac{1}{2 \times 5.1}\left(\frac{50.6}{30}\right)^2 = 548.7 \text{ kip-ft}$$

Since the top flange of the beam is braced by the floor deck, the nominal capacity of the section is the plastic moment capacity ϕM_p. For the W18 × 35 selected for negative moment, Table 7.2 shows $\phi M_p = 180 < 548.7$ kip-ft. Hence this section is not adequate for the maximum positive moment. The least-weight beam with $\phi M_p > 548.7$ kip-ft is a W24 × 84 ($\phi M_p = 605$ kip-ft). If, however, the clearance between the beam and the ceiling does not limit the depth of the beam to 24 in, a W27 × 84 may be preferred; it has greater moment capacity and stiffness.

7.12 COMPOSITE BEAMS

Composite steel beam construction is common in multistory commercial buildings. Utilizing the concrete deck as the top (compression) flange of a steel beam to resist maximum positive moments produces an economical design. In general, composite floorbeam construction consists of the following:

- Concrete over a metal deck, the two acting as one composite unit to resist the total loads. The concrete is normally reinforced with welded wire mesh to control shrinkage cracks.
- A metal deck, usually 1½, 2, or 3 in deep, spanning between steel beams to carry the weight of the concrete until it hardens, plus additional construction loads.
- Steel beams supporting the metal deck, concrete, construction, and total loads. When unshored construction is specified, the steel beams are designed as noncomposite to carry the weight of the concrete until it hardens, plus additional construction loads. The steel section must be adequate to resist the total loads acting as a composite system integral with the floor slab.
- Shear connectors, studs, or other types of mechanical shear elements welded to the top flange of the steel beam to ensure composite action and to resist the horizontal shear forces between the steel beam and the concrete deck.

The effective width of the concrete deck as a flange of the composite beam is defined in Art. 6.26.1. The compression force C (kips) in the concrete is the smallest of the values given by Eqs. (7.24) to (7.26). Equation (7.24) denotes the design strength of the concrete:

$$C_c = 0.85 f'_c A_c \tag{7.24}$$

where f'_c = concrete compressive strength, ksi
A_c = area of the concrete within the effective slab width, in^2 (If the metal deck ribs are perpendicular to the beam, the area consists only of the concrete above the metal deck. If, however, the ribs are parallel to the beam, all the concrete, including the concrete in the ribs, comprises the area.)

Equation (7.25) gives the yield strength of the steel beam:

$$C_t = A_s F_y \tag{7.25}$$

where A_s = area of the steel section (not applicable to hybrid sections), in^2
F_y = yield strength of the steel, ksi

Equation (7.26) expresses the strength of the shear connectors:

$$C_s = \Sigma Q_n \tag{7.26}$$

where ΣQ_n is the sum of the nominal strength of the shear connectors between the point of maximum positive moment and zero moment on either side.

For full composite design. three locations of the plastic neutral axis are possible. The location depends on the relationship of C_c to the yield strength of the web, $P_{yw} = A_w F_y$, and C_t. The three cases are as follows (Fig. 7.5):

Case 1. The plastic neutral axis is located in the web of the steel section. This case occurs when the concrete compressive force is less than the web force $C_c \leq P_{yw}$.

Case 2. The plastic neutral axis is located within the thickness of the top flange of the steel section. This case occurs when $P_{yw} < C_c < C_t$.

Case 3. The plastic neutral axis is located in the concrete slab. This case occurs when $C_c \geq C_t$. (When the plastic axis occurs in the concrete slab, the tension in the concrete below the plastic neutral axis is neglected.)

The AISC ASD and LRFD "Specification for Structural Steel Buildings" restricts the number of studs in one rib of metal deck perpendicular to the axis of beam to three. Maximum spacing along the beam is 36 in $\leq 8t$, where t = total slab thickness (in). When the metal deck ribs are parallel to the axis of the beam, the number of rows of studs depends on the flange width of the beam.

The minimum spacing of studs is six diameters along the longitudinal axis of the beam (4½ in for ¾-in-diameter studs) and four diameters transverse to the beam (3 in for ¾-in-diameter studs).

The total horizontal shear force C at the interface between the steel beam and the concrete slab is assumed to be transmitted by shear connectors. Hence the number of shear connectors required for composite action is

$$N_s = C/Q_n \tag{7.27}$$

where Q_n = nominal strength of one shear connector, kips
N_s = number of shear studs between maximum positive moment and zero moment on each side of the maximum positive moment.

The nominal strength of a shear stud connector embedded in a solid concrete slab may be computed from

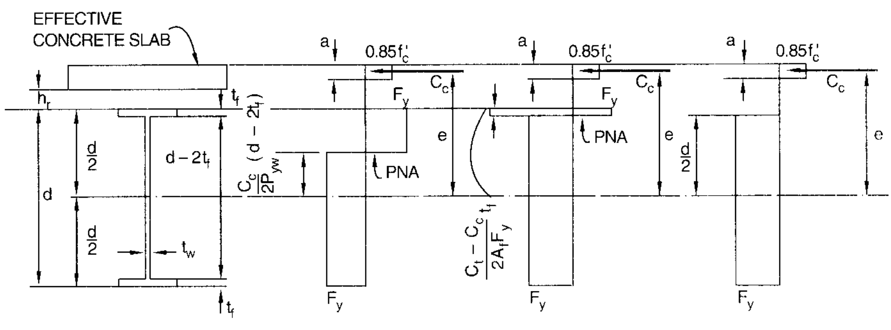

FIGURE 7.5 Stress distributions assumed for plastic design of a composite beam. (*a*) Cross section of composite beam. (*b*) Plastic neutral axis (PNA) in the web. (*c*) PNA in the steel flange. (*d*) PNA in the slab.

$$Q_n = 0.5A_{sc}\sqrt{f'_c E_c} \tag{7.28}$$

where A_{sc} = cross-sectional area of stud, in^2
f'_c = specified compressive strength of concrete, ksi
E_c = modulus of elasticity of the concrete, ksi
= $w^{1.5}\sqrt{f'_c}$
w = unit weight of the concrete, lb/ft^3

The strength Q_n, however, may not exceed $A_{sc}F_u$, where F_u is minimum tensile strength of a stud (ksi).

When the shear connectors are embedded in concrete on a metal deck, a reduction factor R should be applied to Q_n computed from Eq. (7.28).

When the ribs of the metal deck are perpendicular to the beam,

$$R = \frac{0.85}{\sqrt{N_r}}\frac{w_r}{h_r}\left(\frac{H_s}{h_r} - 1\right) \leq 1 \tag{7.29}$$

where N_r = number of studs in one rib at a beam, not to exceed 3 in computations
w_r = average width of concrete rib or haunch, at least 2 in but not more than the minimum clear width near the top of the steel deck, in
h_r = nominal rib height, in
H_s = length of stud in place but not more than $h_r + 3$ in computations, in

When the ribs of the steel deck are parallel to the steel beam and $w_r/h_r < 1.5$, a reduction factor R should be applied to Q_n computed from Eq. (7.28):

$$R = 0.6\,\frac{w_r}{h_r}\left(\frac{H_s}{h_r} - 1\right) \leq 1 \tag{7.30}$$

For this orientation of the deck ribs, the average width w_r should be at least 2 in for the first stud in the transverse row plus four stud diameters for each additional stud.

For a beam with nonsymmetrical loading, the distances between the maximum positive moment and point of zero moment (inflection point) on either side of the point of maximum moment will not be equal. Or, if one end of a beam has negative moment, then the inflection point will not be at that end.

When a concentrated load occurs on a beam, the number of shear connectors between the concentrated load and the inflection point should be adequate to develop the maximum moment at the concentrated load.

When the moment capacity of a fully composite beam is much greater than the applied moment, a partially composite beam may be utilized. It requires fewer shear connectors and thus has a lower construction cost. A partially composite design also may be used advantageously when the number of shear connectors required for a fully composite section cannot be provided because of limited flange width and length.

Figure 7.6 shows seven possible locations of the plastic neutral axis (PNA) in a steel section. The horizontal shear between the steel section and the concrete slab, which is equal to the compressive force in the concrete C, can be determined as illustrated in Table 7.3.

7.13 LRFD FOR COMPOSITE BEAM WITH UNIFORM LOADS

The typical floor construction of a multistory building is to have composite framing. The floor consists of 3¼-in-thick lightweight concrete over a 2-in-deep steel deck. The concrete weighs 115 lb/ft^3 and has a compressive strength of 3.0 ksi. An additional 30% of the dead load is assumed for equipment load during construction. The deck is to be supported on

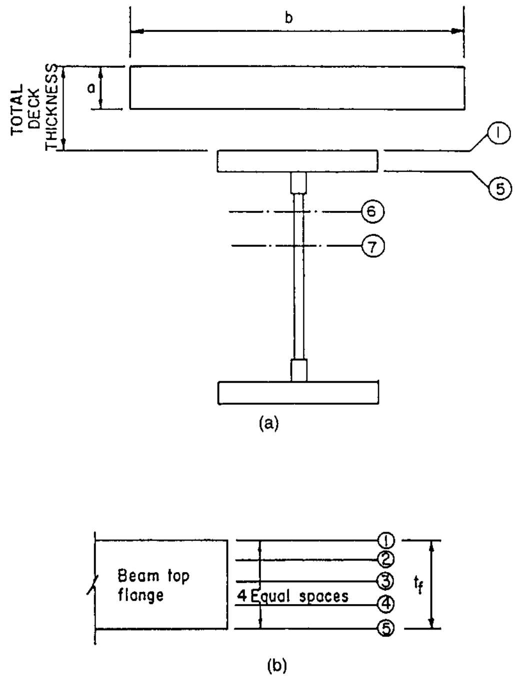

FIGURE 7.6 Seven locations of the plastic neutral axis used for determining the strength of a composite beam. (*a*) For cases 6 and 7, the PNA lies in the web. (*b*) For cases 1 through 5, the PNA lies in the steel flange.

steel beams with stud shear connectors on the top flange for composite action (Art. 7.12). Unshored construction is assumed. Therefore, the beams must be capable of carrying their own weight, the weight of the concrete before it hardens, deck weight, and construction loads. Shear connectors will be ¾ in in diameter and 3½ in long. The floor system should be investigated for vibration, assuming a damping ratio of 5%.

A typical beam supporting the deck is 30 ft long. The distance to adjacent beams is 10 ft. Ribs of the deck are perpendicular to the beam. Uniform dead loads on the beam are

TABLE 7.3 Q_n for Partial Composite Design (kips)

Location of PNA	Q_n and concrete compression
(1)	A_sF_y
(2) to (5)	$A_sF_y - 2\Delta A_f F_y$*
(6)	$0.5[C(5) + C(7)]$†
(7)	$0.25A_sF_y$

* ΔA_f = area of the segment of the steel flange above the plastic neutral axis (PNA).

† $C(n)$ = compressive force at location (n).

construction, 0.50 kips per ft, plus 30% for equipment loads, and superimposed load, 0.25 kips per ft. Uniform live load is 0.50 kips per ft.

Beam Selection. Initially, a beam of A36 steel that can support the construction loads is selected. It is assumed to weigh 26 lb/ft. Thus the beam is to be designed for a service dead load of $0.5 \times 1.3 + 0.026 = 0.676$ kips per ft.

$$\text{Factored load} = 0.676 \times 1.4 = 0.946 \text{ kips per ft}$$

$$\text{Factored moment} = M_u = 0.946 \times 30^2/8 = 106.5 \text{ kip-ft}$$

The plastic section modulus required therefore is

$$Z = \frac{M_u}{\phi F_y} = \frac{106.5 \times 12}{0.9 \times 36} = 39.4 \text{ in}^3$$

Use a W16 × 26 ($Z = 44.2$ in^3 and moment of inertia $I = 301$ in^4).

The beam should be cambered to offset the deflection due to a dead load of $0.50 + 0.026 = 0.526$ kips per ft.

$$\text{Camber} = \frac{5 \times 0.526 \times 30^4 \times 12^3}{384 \times 29{,}000 \times 301} = 1.1 \text{ in}$$

Camber can be specified on the drawings as 1 in.

Strength of Fully Composite Section. Next, the composite steel section is designed to support the total loads. The live load may be reduced in accordance with area supported (Art. 7.9). The reduction factor is $R = 0.0008(300 - 150) = 0.12$. Hence the reduced live load is $0.5(1 - 0.12) = 0.44$ kips per ft. The factored load is the larger of the following:

$$1.2(0.50 + 0.25 + 0.026) + 1.6 \times 0.44 = 1.635 \text{ kips per ft}$$

$$1.4(0.5 + 0.25 + 0.026) = 1.086 \text{ kips per ft}$$

Hence the factored moment is

$$M_u = 1.635 \times 30^2/8 = 183.9 \text{ kip-ft}$$

The concrete-flange width is the smaller of $b = 10 \times 12 = 120$ in or $b = 2(30 \times 12/8) = 90$ in (governs).

The compressive force in the concrete C is the smaller of the values computed from Eqs. (7.24) and (7.25).

$$C_c = 0.85 f'_c A_c = 0.85 \times 3 \times 90 \times 3.25 = 745.9 \text{ kips}$$

$$C_t = A_s F_y = 7.68 \times 36 = 276.5 \text{ kips (governs)}$$

The depth of the concrete compressive-stress block (Fig. 7.5) is

$$a = \frac{C}{0.85 f'_c \text{b}} = \frac{276.5}{0.85 \times 3.0 \times 90} = 1.205 \text{ in}$$

Since $C_c > C_t$, the plastic neutral axis will line in the concrete slab (case 3, Art. 7.12). The distance between the compression and tension forces on the W16 × 26 (Fig. 7.5*d*) is

$$e = 0.5d + 5.25 - 0.5a$$
$$= 0.5 \times 15.69 + 5.25 - 0.5 \times 1.205 = 12.493 \text{ in}$$

The design strength of the W16 × 26 is

$$\phi M_n = 0.85 C_t e = 0.85 \times 276.5 \times 12.493/12 = 244.7 \text{ kip-ft} > 183.9 \text{ kip-ft—OK}$$

Partial Composite Design. Since the capacity of the full composite section is more than required, a partial composite section may be satisfactory. Seven values of the composite section (Fig. 7.6) are calculated as follows, with the flange area $A_f = 5.5 \times 0.345 = 1.898$ in^2.

1. Full composite:

$$\Sigma Q_n = A_s F_y = 276.5 \text{ kips}$$
$$\phi M_n = 244.7 \text{ kip-ft}$$

2. Plastic neutral axis $\Delta A_f = A_f/4 = 0.4745$ in below the top of the top flange. From Table 7.3, $\Sigma Q_n = A_s F_y - 2\Delta A_f F_y$.

$$\Sigma Q_n = 276.5 - 2 \times 0.4745 \times 36 = 242.3$$
$$a = 242.3/(0.85 \times 3.0 \times 90) = 1.0558 \text{ in}$$
$$e = 15.69/2 + 5.25 - 1.0558/2 = 12.567 \text{ in}$$
$$M_n = 242.3 \times 12.567 + 0.5(276.5 - 242.3)$$
$$\times \left((15.69 - 0.345 \frac{276.5 - 242.3}{2 \times 1.898 \times 36}\right)$$
$$= 3{,}312 \text{ kip-in}$$
$$\phi M_n = 0.85 \times 3312/12 = 234.6 \text{ kip-ft}$$

3. PNA $\Delta Af = A_f/2 = 0.949$ in below the top of the top flange:

$$\Sigma Q_n = 208.2 \text{ kips}$$
$$\phi M_n = 224.0 \text{ kip-ft}$$

4. PNA $\Delta A_f = 3A_f/4 = 1.4235$ in below the top of the top flange:

$$\Sigma Q_n = 174.0 \text{ kips}$$
$$\phi M_n = 212.8 \text{ kip-ft}$$

5. PNA at the bottom of the top flange ($\Delta A_f = A_f$):

$$\Sigma Q_n = 139.9 \text{ kips}$$
$$\phi M_n = 201.0 \text{ kip-ft}$$

6. Plastic neutral axis within the web. ΣQ_n is the average of items 5 and 7. (See Table 7.3.)

$$\Sigma Q_n = (139.9 + 69.1)/2 = 104.5 \text{ kips}$$

$$\phi M_n = 186.4 \text{ kip-ft}$$

7.

$$\Sigma Q_n = 0.25 \times 276.5 = 69.1 \text{ kips}$$

$$\phi M_n = 166.7 \text{ kip-ft}$$

From the partial composite values 2 to 7, value 6 is just greater than $M_u = 183.9$ kip-ft.

The AISC "Manual of Steel Construction" includes design tables for composite beams that greatly simplify the calculations. For example, the table for the W16 $\times$ 26, grade 36, composite beam gives ϕM_n for the seven positions of the PNA and for several values of the distance Y_2 (in) from the concrete compressive force C to the top of the steel beam. For the preceding example,

$$Y_2 = Y_{con} - a/2 \tag{7.31}$$

where Y_{con} = total thickness of floor slab, in
a = depth of the concrete compressive-stress block, in

From the table for case 6, $\Sigma Q_n = 104$ kips.

$$a = \frac{104}{0.85 \times 3.0 \times 90} = 0.453 \text{ in}$$

Substitution of a and $Y_{con} = 5.25$ in in Eq. (7.31) gives

$$Y_2 = 5.25 - 0.453/2 = 5.02 \text{ in}$$

The manual table gives the corresponding moment capacity for case 6 and $Y_2 = 5.02$ in as

$$\phi M_n = 186 \text{ kip-ft} > 183.9 \text{ kip-ft—OK}$$

The number of shear studs is based on $C = 104.5$ kips. The nominal strength Q_n of one stud is given by Eq. (7.28). For a ¾-in stud, with shearing area $A_{sc} = 0.442$ in^2 and tensile strength $F_u = 60$ ksi, the limiting strength is $A_{sc}F_u = 0.442 \times 60 = 26.5$ kips. With concrete unit weight $w = 115$ lb/ft^3 and compressive strength $f'_c = 3.0$ ksi, and modulus of elasticity $E_c = 2136$ ksi, the nominal strength given by Eq. (7.28) is

$$Q_n = 0.5 \times 0.442 \sqrt{3.0 \times 2136} = 17.7 \text{ kips} < 26.5 \text{ kips}$$

The number of shear studs required is $2 \times 104.5/17.7 = 11.8$. Use 12. The total number of metal deck ribs supported on the steel beam is 30. Therefore, only one row of shear studs is required, and no reduction factor is needed.

Deflection Calculations. Deflections are calculated based on the partial composite properties of the beam. First, the properties of the transformed full composite section (Fig. 7.7) are determined.

The modular ratio E_s/E_n is $n = 29{,}000/2136 = 13.6$. This is used to determine the transformed concrete area $A_1 = 3.25 \times 90/13.6 = 21.52$ in^2. The area of the W16 $\times$ 26 is 7.68 in^2, and its moment of inertia $I_s = 301$ in^4. The location of the elastic neutral axis is determined by taking moments of the transformed concrete area and the steel area about the top of the concrete slab:

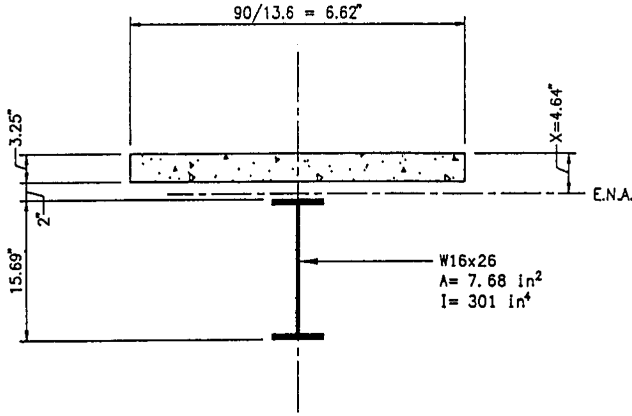

FIGURE 7.7 Transformed section of a composite beam.

$$X = \frac{21.52 \times 3.25/2 + 7.68(0.5 \times 15.69 + 5.25)}{21.52 + 7.68} = 4.64 \text{ in}$$

The elastic transformed moment of inertia for full composite action is

$$I_{tr} = \frac{90 \times 3.25^3}{13.6 \times 12} + 21.52\left(4.64 - \frac{3.25}{2}\right)^2 + 7.68\left(\frac{15.69}{2} + 5.25 - 4.64\right)^2 + 301$$
$$= 1065 \text{ in}^4$$

Since partial composite construction is used, the effective moment of inertia is determined from

$$I_{eff} = I_s + (I_{tr} - I_s)\sqrt{\Sigma Q_n / C_f} \tag{7.32}$$

where C_f = concrete compression force based on full composite action

$$I_{eff} = 301 + (1065 - 301)\sqrt{104.5/276.5} = 770.7 \text{ in}^4$$

I_{eff} is used to calculate the immediate deflection under service loads (without long-term effects).

For long-term effect on deflections due to creep of the concrete, the moment of inertia is reduced to correspond to a 50% reduction in E_c. Accordingly, the transformed moment of inertia with full composite action and 50% reduction in E_c is I_{tr} = 900.3 in^4 and is based on a modular ratio $2n$ = 27.2. The corresponding transformed concrete area is A_1 = 10.76 in^2.

The reduced effective moment of inertia for partial composite construction with long-term effect is determined from Eq. (7.32):

$$I_{eff} = 301 + (900.3 - 301)\sqrt{104.5/276.5} = 669.4 \text{ in}^4$$

Since unshored construction is specified, the deflection under the weight of concrete when placed and the steel weight is compensated for by the camber specified. Long-term effect due to these weights need not be considered because the concrete is not stressed by them.

Deflection due to long-term superimposed dead loads is

$$D_1 = \frac{5 \times 0.25 \times 30^4 \times 12^3}{384 \times 29{,}000 \times 669.4} = 0.235 \text{ in}$$

Deflection due to short-term (reduced) live load is

$$D_2 = \frac{5 \times 0.44 \times 30^4 \times 12^3}{384 \times 29{,}000 \times 770.7} = 0.358 \text{ in}$$

Total deflection is

$$D = D_1 + D_2 = 0.235 + 0.358 = 0.593 \text{ in} = L/607\text{—OK}$$

Vibration Investigation. The vibration study of composite beams is based on Wiss and Parmlee's "drop-of-the-heel" method and Murray's empirical equation. (T. M. Murray, "Design to Prevent Floor Vibrations," *AISC Engineering Journal,* third quarter, 1979, and "Acceptability Criterion for Occupant-Induced Floor Vibrations," *AISC Engineering Journal,* second quarter, 1989. Also see T. M. Murray et al, "Floor Vibrations due to Human Activity," *AISC Steel Design Guide* No. 11, 1997.)

The total dead load W_D considered in the vibration equations consists of the weight of the concrete and steel beam plus a percentage of the superimposed dead load. The percentage of superimposed dead load is 30% in this example:

$$\begin{aligned} W_D &= 0.50 \times 30 + 0.026 \times 30 + 0.30 \times 0.25 \times 30 \\ &= 18.0 \text{ kips} \end{aligned}$$

The frequency f (Hz) of a composite simple-span beam is given by

$$f = 1.57 \sqrt{\frac{gEI_t}{W_D L^3}} \tag{7.33}$$

where g = gravitational acceleration = 386.4 in/s^2
E = steel modulus of elasticity, ksi
I_t = transformed moment of inertia of the composite section, in^4
W_D = total weight on the beam, kips
L = span, in

Substitution of previously determined values into Eq. (7.33) yields

$$f = 1.57 \sqrt{\frac{386.4 \times 29{,}000 \times 770.7}{18.0(30 \times 12)^3}} = 5.03 \text{ Hz}$$

The amplitude A_o of a single beam is calculated by dividing the total floor amplitude A_{ot} by the number N_{eff} of effective beams:

$$A_o = A_{ot}/N_{eff} \tag{7.34}$$

For a constant $t_o = (1/\pi f) \tan^{-1} a \leq 0.05$,

$$A_{ot} = 0.246L^3(0.10 - t_o)/EI_t \tag{7.35}$$

For $t_o > 0.05$,

$$A_{ot} = \frac{0.246L^3}{EI_t} \times \frac{1}{2\pi f} \sqrt{2(1 - a \sin a - \cos a) + a^2} \tag{7.36}$$

where $a = 0.1\pi f = 0.1\pi \times 5.03 = 1.58$ radians.

The number of effective beams can be determined from

$$N_{eff} = 2.967 - 0.05776(S/d_e) + 2.556 \times 10^{-8}L^4/I_t + 0.0001(L/S)^3 \geq 1.0 \qquad (7.37)$$

where S = spacing of beams in the floor, in
d_c = effective depth of the slab, in
= average slab thickness when the metal deck ribs are perpendicular to the beam
= concrete thickness above the metal deck when the deck ribs are parallel to the beam

With $S = 120$ in and $d_c = 4.25$ in, the number of effective beams is

$$N_{eff} = 2.967 - 0.05776 \times \frac{120}{4.25} + 2.556 \times 10^{-8} \times \frac{360^4}{770.7} + 0.0001\left(\frac{360}{120}\right)^3 = 1.90$$

For $t_o = (1/1.58\pi)\tan^{-1} 1.58 > 0.05$, the total floor amplitude is, from Eq. (7.36),

$$\begin{aligned} A_{ot} &= \frac{0.246 \times 360^3}{29{,}000 \times 770.7} \times \frac{1}{2\pi \times 5.04} \\ &\quad \times \sqrt{2(1 - 1.58 \sin 1.58 - \cos 1.58) + 1.58^2} \\ &= 0.188 \text{ in} \end{aligned}$$

The amplitude of one beam then is, by Eq. (7.34),

$$A_o = A_{ot}/N_{eff} = 0.0188/1.9 = 0.0099 \text{ in}$$

The mean response rating is given by

$$R = 5.08\left(\frac{fA_o}{D^{0.217}}\right)^{0.265} \qquad (7.38)$$

where f = frequency of the composite beam, Hz
A_o = maximum amplitude of one beam, in
D = damping ratio

For the following values of R, the rating denotes

1. Imperceptible vibration
2. Barely perceptible vibration
3. .Distinctly perceptible vibration
4. Strongly perceptible vibration
5. Severe vibration

For the assumed 5% damping ratio,

$$R = 5.08\left(\frac{5.03 \times 0.0099}{0.05^{0.217}}\right)^{0.265} = 2.7$$

For $2.5 < 3.5$, the vibration may be considered distinctly perceptible.

Murray's equation gives the minimum acceptable damping (percent) as

$$D = 35A_of + 2.5 = 35 \times 0.0099 \times 5.03 + 2.5 = 4.2 < 5.0 \text{ (acceptable)}$$

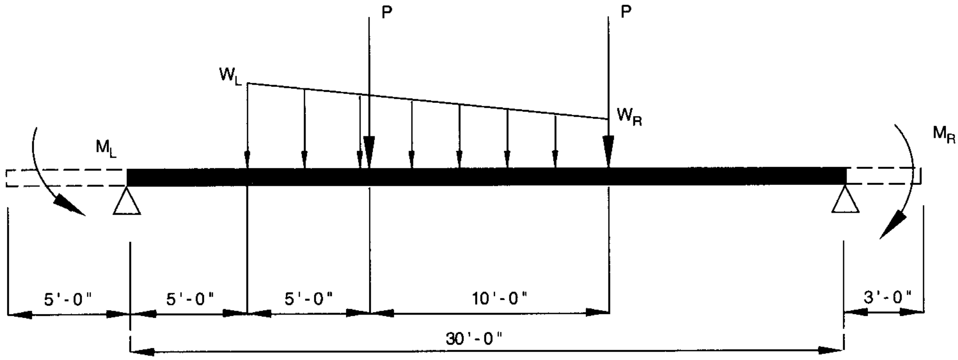

FIGURE 7.8 Composite beam with overhang carries two concentrated loads and a uniformly decreasing load over part of the span. Cantilever carries uniform loads.

7.14 EXAMPLE—LRFD FOR COMPOSITE BEAM WITH CONCENTRATED LOADS AND END MOMENTS

The general information for design of a floor system is the same as that given in Art. 7.14. In this example, a girder of grade 50 steel is to support the floorbeams. (Deck ribs are parallel to the girder.) The girder loads and span are shown in Fig. 7.8 and Table 7.4. The spacing to the left adjacent girder is 30 ft and to the right girder 20 ft.

Dead-Load Moment for Unshored Beam. The steel girder is to support construction dead loads, nonshored, with 30% additional dead load assumed applied during construction. The girder is assumed to weigh 44 lb/ft. The negative end moments are neglected for this phase of the design since the concrete may be placed over the entire span between the supports but not over the cantilever.

The factored dead loads are

$$P_u = 14.85 \times 1.30 \times 1.4 = 27.03 \text{ kips}$$

$$W_{Lu} = 0.5 \times 1.30 \times 1.4 = 0.910 \text{ kips per ft}$$

$$W_{Ru} = 0.2 \times 1.30 \times 1.4 = 0.364 \text{ kips per ft}$$

$$W_{Gu} = 0.044 \times 1.4 = 0.062 \text{ kips per ft}$$

For the girder acting as a simple beam with a 30-ft span, the factored dead-load moment is

TABLE 7.4 Concentrated and Partial Loads on Composite Beam

Type of load	Construction dead load	Superimposed dead load	Live load
Concentrated load P, kips	14.85	7.5	15.0
Negative moment M_L, kip-ft	22.5	7.5	20.0
Negative moment M_R, kip-ft	7.5	2.5	7.0
Partial-load start w_L, kips per ft	0.50	0.75	0.50
Partial-load end w_R, kips per ft	0.20	0.30	0.20

$M_u = 328.0$ ft-kips, and the plastic modulus required is $Z = M_u/0.9F_y = 328 \times 12/(0.9 \times 50) = 87.5$ in^3. The least-weight section with larger modulus is a W21 $\times$ 44, with $Z = 95.4$ in^3.

Camber. This is computed for maximum deflection attributable to full construction dead loads. For this computation, the dead-load portion of the end moments is included. The loads are listed under construction dead load in Table 7.4.

The corresponding deflection is 1.09 in. A camber of 1 in may be specified.

Design for Maximum End Moment. This takes into account the unbraced length of the girder. For the maximum possible unbraced length of the bottom (compression) flange of the steel section, only the dead loads act between supports. The factored dead loads are

$$P_u = 1.2 \times 14.85 = 17.82 \text{ kips}$$

$$W_{Lu} = 1.2 \times 0.5 = 0.60 \text{ kips per ft}$$

$$W_{Ru} = 1.2 \times 0.2 = 0.24 \text{ kips per ft}$$

$$W_{Gu} = 1.2 \times 0.044 = 0.053 \text{ kips per ft}$$

$$M_{Lu} = 1.2(22.5 + 7.5) + 1.6 \times 20 = 68.0 \text{ kip-ft}$$

$$M_{Ru} = 1.2(7.5 + 2.50) + 1.6 \times 7 = 23.2 \text{ kip-ft}$$

The unbraced length of the bottom flange is 2.9 ft. The cantilever length is 5 ft (governs). The design strength ϕM_n for a wide-flange section of grade 50 steel may be obtained from curves in the AISC "Steel Construction Manual—LRFD." A curve indicates that the W21 $\times$ 44 with an unbraced length of 5 ft has a design strength $\phi M_n = 356$ kip-ft.

Design for Positive Moment. For this computation, the load factor used for the negative dead-load moments is 1.2, with only dead load on the cantilevers. The load factor for live loads is 1.6.

The factored loads, with live loads reduced 40% for the size of areas supported, are

$$P_u = 1.2(14.85 + 7.5) + 1.6 \times 9.0 = 41.22 \text{ kips}$$

$$W_{Lu} = 1.2(0.5 + 0.75) + 1.6 \times 0.30 = 1.98 \text{ kips per ft}$$

$$W_{Ru} = 1.2\,(0.20 + 0.30) + 1.6 \times 0.12 = 0.792 \text{ kips per ft}$$

$$W_{Gu} = 1.2 \times 0.044 = 0.053 \text{ kips per ft}$$

$$M_{Lu} = 1.2 \times 22.5 = 27.0 \text{ kip-ft}$$

$$M_{Ru} = 1.2 \times 7.5 = 9.0 \text{ kip-ft}$$

For these loads, the factored maximum positive moment is $M_u = 509.6$ kip-ft.

For determination of the capacity of the composite beam, the effective concrete-flange width is the smaller of

$$b = 12(30 + 20)/2 = 300 \text{ in}$$

$$b = 12 \times 30/4 = 90 \text{ in (governs)}$$

Design tables for composite beams in the AISC manual greatly simplify calculation of design strength. For example, the table for the W21 $\times$ 44 grade 50 beam gives ϕM_n for seven positions of the plastic neutral axis (PNA) and for several values of the distance Y_2 from the top of the steel beam to the centroid of the effective concrete-flange force (ΣQ_n) (see Art. 7.13). Try $\Sigma Q_n = 260$ kips. The corresponding depth of the concrete compression block is

$$a = \frac{260}{0.85 \times 3.0 \times 90} = 1.133 \text{ in}$$

From Eq. (7.31), $Y_2 = 5.25 - 1.133/2 = 4.68$ in. The manual table gives the corresponding design strength for case 6 and $Y_2 = 4.68$ in, by interpolation, as

$$\phi M_n = 546 \text{ kip-ft} > (M_u = 509.6 \text{ kip-ft})$$

The maximum positive moment M_u occurs 13.25 ft from the left support (Fig. 7.8). The inflection points occur 0.49 and 0.19 ft from the left and right supports, respectively.

Shear Connectors. Next, the studs required to develop the maximum positive moment and the moments at the concentrated loads are determined. Welded studs ¾ in in diameter are to be used. As in Art. 7.13, the nominal strength of a stud is $Q_n = 17.7$ kips.

For development of the maximum positive moment on both sides of the point of maximum moment, with $\Sigma Q_n = 260$ kips, at least $260/17.7 = 14.69$ studs are required. Since the negative-moment region is small, it is not practical to limit the stud placement to the positive-moment region only. Therefore, additional studs are required for placement of connectors over the entire 30-ft span.

Stud spacing on the left of the point of maximum moment should not exceed

$$S_L = 12(13.25 - 0.49)/14.69 = 10.42 \text{ in}$$

Stud spacing on the right of the point of maximum moment should not exceed

$$S_R = 12(30 - 13.25 - 0.19)/14.69 = 13.53 \text{ in}$$

For determination of the number of studs and spacing required between the concentrated load P 10 ft from the left support (Fig. 7.8) and the left inflection point, the maximum load at that load is calculated to be $M_{Lu} = 502.1$ kip-ft. For the W21 × 44 grade 50 beam, the manual table indicates that for $\Sigma Q_n = 260$ kips and $Y_2 = 4.68$ in, as calculated previously, the design strength is $\phi M_n = 546$ kip-ft. For ¾-in studs and $\Sigma Q_n = 260$ kips, the required number of studs is 14.69. Spacing of these studs, which may not exceed 10.42 in, is also limited to

$$S_{PL} = 12(10 - 0.49)/14.69 = 7.77 \text{ in}$$

Hence the number of studs to be placed in the 10 ft between P and the left support is $10 \times 12/7.77 = 15.4$ studs. Use 16 studs.

For determination of the number of studs and spacing required between the concentrated load P 10 ft from the right support (Fig. 7.8) and the right inflection point, the maximum moment at that load is calculated to be $M_{Ru} = 481.2$ kip-ft. For the W21 × 44, the manual table indicates that, for case 7, $\Sigma Q_n = 163$ kips and $\phi M_n = 486$ kip-ft. The required number of studs for $\Sigma Q_n = 163$ kips is $163/17.7 = 9.21$ studs. Spacing of these studs, which may not exceed 13.53 in, is also limited to

$$S_{PR} = 12(10 - 0.19)/9.21 = 12.78 \text{ in}$$

The number of studs to be placed in the 10 ft between P and the right support is $10 \times 12/12.78$. Use 10 studs.

The number of studs required between the two concentrated loads equals the sum of the number required between the point of maximum moment and P on the left and right. On the left, the required number of studs is $13.25 \times 12/10.42 - 16 = -0.74$. Since the result is negative, use on the left the maximum permissible stud spacing of 36 in. On the right, the required number of studs is $16.75 \times 12/13.53 - 10 = 4.85$. Use 5 studs. The spacing should not exceed $12(16.75 - 10)/5 = 16.2$ in. Specification of one spacing for the middle segment, however, is more practical. Accordingly, the number of studs between the two concentrated loads would be based on the smallest spacing on either side of the point of maximum moment: $10 \times 12/16.2 = 7.4$. Use 8 studs spaced 15 in center to center.

It may be preferable to specify the total number of studs placed on the beam based on one uniform spacing. The spacing required to develop the maximum moment on either side of its location and between each concentrated load and a support is 7.77 in, as calculated previously. For this spacing over the 30-ft span, the total number of studs required is $30 \times 12/7.77 = 46.3$. Use 48 studs (the next even number).

Deflection Computations. The elastic properties of the composite beam, which consists of a W21 × 44 and a concrete slab 5.25 in deep (an average of 4.25 in deep) and 90 in wide, are as follows:

$$E_c = 115^{1.5}\sqrt{3.0} = 2136 \text{ ksi}$$

$$n = E_s/E_c = 29{,}000/2136 = 13.58$$

$$b/n = 90/13.58 = 6.63 \text{ in}$$

$$I_{tr} = 2496 \text{ in}^4$$

For determination of the effective moment of inertia I_{eff} at the location of the maximum moment, a reduced value of the transformed moment of inertia I_{tr} is used based on the partial-composite construction assumed in the computation of shear-connector requirements. For use in Eq. (7.32), the moment of inertia of the W21 × 44 is $I_s = 843$ in^4, $Q_n = 260$ kips, and C_f is the smaller of

$$C_f = 0.85 f'_c A_c = 0.85 \times 3.0 \times 4.25 \times 90 = 975.4 \text{ kips}$$

$$C_f = A_s F_y = 13.0 \times 50 = 650 \text{ kips (governs)}$$

$$I_{eff} = 843 + (2496 - 843)\sqrt{260/650} = 1888 \text{ in}^4$$

A reduced moment of inertia I_r due to long-time effect (creep of the concrete) is determined based on a modular ratio $2n = 2 \times 13.58 = 27.16$ and effective slab width $b/n = 90/27.16 = 3.31$ in. The reduced transformed moment of inertia is 2088 in^4 and the reduced effective moment of inertia is

$$I_r = 843 + (2088 - 843)\sqrt{260/650} = 1630 \text{ in}^4$$

The deflection computations for unshored construction exclude the weight of the concrete slab and steel beam. Whether or not the steel beam is adequately cambered, the assumption is made that the concrete will be finished as a level surface. Hence the concrete slab is likely to be thicker at midspan of the beams and deck.

For computation of the midspan deflections, the cantilevers are assumed to carry only dead load. From Table 7.4, the superimposed dead loads are $P_s = 7.5$ kips, $w_{Ls} = 0.75$ kips per ft, and $w_{Rs} = 0.30$ kips per ft. The dead-load end moments are $M_L = 22.5$ kip-ft and $M_R = 7.5$ kip-ft. For $I_r = 1630$ in^4, the maximum deflection due to these loads is

$$D = \frac{15{,}865{,}000}{29{,}000 \times 1630} = 0.336 \text{ in}$$

The deflection at the left concentrated load P is 0.296 in and at the second load, 0.288 in.

From Table 7.4, the live loads with a 40% reduction for size of area supported are P_L = 9.0 kips, w_{LL} = 0.30 kips per ft, and w_{RL} = 0.12 kips per ft. The maximum deflection due to these loads and with an effective moment of inertia of 1888 in^4 is 0.319 in. The deflection at the left load is 0.282 in and at the second load, 0.275 in.

Total deflections due to superimposed dead loads and live loads are

$$\text{Maximum deflection} = 0.336 + 0.319 = 0.655 \text{ in}$$

$$\text{Deflection at left load } P = 0.295 + 0.282 = 0.577 \text{ in}$$

$$\text{Deflection at right load } P = 0.288 + 0.275 = 0.563 \text{ in}$$

7.15 COMBINED AXIAL LOAD AND BIAXIAL BENDING

Members subject to axial compression or tension and bending about one or two axes, such as columns that are part of rigid frames in two directions, are designed to satisfy the following interaction equations. For symmetrical shapes when $P_u/\phi P_n \geq 0.2$,

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \qquad (7.39a)$$

For $P_u/\phi P_n < 0.2$,

$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \qquad (7.39b)$$

where P_u = factored axial load, kips
P_n = nominal compressive or tensile strength, kips
M_u = factored bending moment, kip-in
M_n = nominal flexural strength, kip-in
ϕ_t = resistance factor for tension = 0.90
ϕ_b = resistance factor for flexure = 0.90

The factored moments M_{ux} and M_{uy} should include second-order effects, such as $P - \Delta$, for the factored loads. If second-order analysis is not performed, the factored moments can be calculated with magnifiers as follows:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \qquad (7.40)$$

where M_{nt} = factored bending moment based on the assumption that there is no lateral translation of the frame, kip-in
M_{lt} = factored bending moment as a result only of lateral translation of the frame, kip-in
$B_1 = \dfrac{C_m}{(1 - P_u/P_e)} \geq 1$
$P_e = A_g F_y / \lambda_c^2$
λ_c = slenderness parameter (Art. 7.4) with effective length factor $K \leq 1.0$ in the plane of bending
A_g = gross area of the member, in^2
F_y = specified minimum yield point of the member, ksi
C_m = coefficient defined for Eq. (6.67)

$$B_2 = \frac{1}{1 - \Sigma P_u \dfrac{\Delta_{oh}}{\Sigma HL}} \text{ or } \frac{1}{1 - \dfrac{\Sigma P_u}{\Sigma P_e}}$$

ΣP_u = sum of factored axial loads of all columns in a story, kips
Δ_{oh} = translation deflection of the story under consideration, in
ΣH = sum of all horizontal forces in a story that produce Δ_{oh}, kips
L = story height, in
P_e = $A_g F_y / \lambda_c^2$, where λ_c is the slenderness parameter with the effective length factor $K \geq 1.0$ in the plane of bending determined for the member when sway is permitted, kips

Since several computer analysis programs are available with the $P - \Delta$ feature included, it is advisable to determine the $P - \Delta$ effects by second-order analysis of framing subject to lateral loads. If the $P - \Delta$ effect is evaluated for frames subject to lateral as well as to vertical loads, the moment magnifier B_2 can be considered to be unity.

7.16 EXAMPLE—LRFD FOR WIDE-FLANGE COLUMN IN A MULTISTORY RIGID FRAME

Columns at the ninth level of a multistory building are to be part of a rigid frame that resists wind loads. Typical floor-to-floor height is 13 ft.

In the ninth story, a wide-flange column of grade 50 steel is to carry loads from a transfer girder, which supports an offset column carrying the upper levels. Therefore, the lower column discontinues at the ninth level. The loads on that column are as follows: dead load, 750 kips; superimposed dead load, 325 kips; and live load, 250 kips. The moments due to gravity loads at the beam-column connection are

Dead-load major-axis moment = 180 kip-ft

Live-load major-axis moment = 75 kip-ft

Dead-load minor-axis moment = 75 kip-ft

Live-load minor-axis moment = 40 kip-ft

The column axial loads and moments due to service lateral loads with $P - \Delta$ effect included are

Axial load = 600 kips

Major-axis moment = 1050 kip-ft

Minor-axis moment = 0.0

The beams attached to the flanges of the column with rigid welded connections are part of the rigid frame and have spans of 30 ft. The following beam sizes and corresponding stiffnesses, at top and bottom ends of the column apply.

The beams at both sides of the column at the floor above and the floor below are W36 × 300. The sum of the stiffnesses I_b / L_b of the beams is

$$\Sigma(I_b / L_b) = 20{,}300 \times 2/(30 \times 12) = 112.8 \text{ in}^3$$

where I_b is the beam moment of inertia (in^4).

The effective length factor K_x corresponding to the case of frame with sidesway permitted is used in determining the axial-load capacity and the moment magnifier B_1. The moment magnifier B_2 is considered unity inasmuch as the $P - \Delta$ effect is included in the analysis.

Axial-Load Capacity. Since the column is part of a wind-framing system, the K values should be computed based on column and beam stiffnesses. To determine the major-axis K_x, assume that a W14 × 426 with $I_{cx} = 6600$ in^4 will be selected for the column. At the top of the column, where there is no column above the floor, the relative column-beam stiffness is

$$G_A = \frac{\Sigma(I_c/L_c)}{\Sigma(I_b/L_b)} = \frac{6600/12(13-3)}{112.8} = 0.49$$

At the column bottom, with a W14 × 426 column below,

$$G_B = \frac{\Sigma(I_c/L_c)}{\Sigma(I_b/L_b)} = \frac{2 \times 6600/12(13-3)}{112.8} = 0.98$$

From a nomograph for the case when sidesway is permitted (Fig. 7.9*b*), $K_x = 1.23$ (at the intersection with the K axis of a straight line connecting 0.49 on the G_A axis with 0.98 on the G_B axis).

Since the connection of beams to the column web is a simple connection with inhibited sidesway, $K_y = 1.0$.

The effective lengths to be used for determination of axial-load capacity are

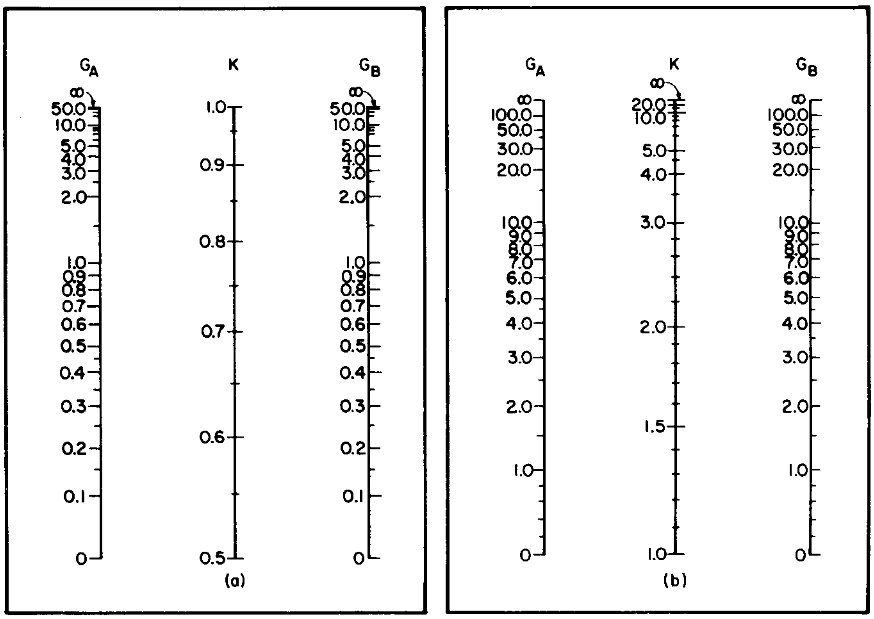

FIGURE 7.9 Nomographs for determination of the effective length factor for a column. (*a*) For use when sidesway is prevented. (*b*) For use when sidesway may occur.

$$K_xL_x = 1.23(13 - 3) = 12.3 \text{ ft}$$

$$K_yL_y = 1.0 \times 13 = 13 \text{ ft}$$

The W14 × 426 has radii of gyration $r_x = 7.26$ in and $r_y = 4.34$ in. Therefore, the slenderness ratios for the column are

$$K_xL_x/r_x = 12.3 \times 12/7.26 = 20.3$$

$$K_yL_y/r_y = 13 \times 12/4.34 = 35.9 \text{ (governs)}$$

Use of the AISC ''Manual of Steel Construction—LRFD'' tables for design axial strength of compression members simplifies evaluation of the trial column size. For the W14 × 426, grade 50 section, a table indicates that for $K_yL_y = 13$ ft, $\phi P_n = 4830$ kips.

Moment Capacity. Next, the nominal bending-moment capacities are calculated. For strong-axis bending moment, $K_yL_y = 13$ ft is assumed for the flange lateral- buckling state. The limiting lateral unbraced length L_p (in) for plastic behavior for the W14 × 426 is

$$L_p = 300r_y/\sqrt{F_y} = 300 \times 4.34/\sqrt{50} = 184 \text{ in } 15.3 \text{ ft} > 13 \text{ ft}$$

Since the unbraced length is less than L_p,

$$\phi M_{nx} = 0.9 \times 869 \times 50/12 = 3259 \text{ kip-ft}$$

$$\phi M_{ny} = 0.9Z_yF_y = 0.9 \times 434 \times 50/12 = 1628 \text{ kip-ft}$$

Interaction Equation for Dead Load. For use in the interaction equation for axial load and bending [Eq. (7.39*a*) or (7.39*b*)], the factored dead load is

$$P_u = 1.4(750 + 325 + 0.426 \times 13) = 1513 \text{ kips}$$

The factored moments applied to columns due to any general loading conditions should include the second-order magnification. When the frame analysis does not include second-order effects, the factored column moment can be determined from Eq. (7.40).

Computer analysis programs usually include the second-order analysis ($P - \Delta$ effects). Therefore, the values of B_2 for moments about both column axes can be assumed to be unity. However, B_1 should be determined for evaluation of the nonsway magnifications. For a braced column (drift prevented), the slenderness coefficient K_x is determined from Fig. 7.9*a* with $G_A = 0.49$ and $G_B = 0.98$, previously calculated. The nomograph indicates that $K_s = 0.73$.

For determination of B_1 in Eq. (7.40), the column when loaded is assumed to have single curvature with end moments $M_1 = M_2$. Hence $C_m = 1$.

For determination of the elastic buckling load P_{ex}, the slenderness parameter is

$$\lambda_{cx} = \frac{KL_x}{r_x\pi}\sqrt{\frac{F_y}{E}} = \frac{KL_x}{r_x}\sqrt{\frac{F_y}{286{,}220}}$$

$$= \frac{0.73 \times 12(13 - 10)}{7.26}\sqrt{\frac{50}{286{,}220}} = 0.159$$

and the elastic buckling load for the beam cross-sectional area $A_g = 125$ in^2 is

$$P_{ex} = A_gF_y/\lambda_{cx}^2 = 125 \times 50/0.159^2 = 247{,}000 \text{ kips}$$

With these values, the magnification factor for M_{ux} is

$$B_{1x} = \frac{C_m}{1 - P_u/P_{ex}} = \frac{1.0}{1 - 1513/247{,}000} = 1.006$$

For determination of the elastic buckling load P_{ey},

$$\lambda_{cy} = \frac{1 \times 13 \times 12}{4.34} \sqrt{\frac{50}{286{,}220}} = 0.475$$

The elastic buckling load with respect to the y axis is

$$P_{ey} = A_g F_y / \lambda_{cy}^2 = 125 \times 50/0.475^2 = 27{,}700 \text{ kips}$$

With these values, the magnification factor for M_{uy} is

$$B_{1y} = \frac{C_m}{1 - P_u/P_{ey}} = \frac{1}{1 - 1513/27{,}700} = 1.058$$

Application of the magnification factor to the dead-load moments due to gravity loads yields

$$M_{ux} = 1.006 \times 1.4 \times 180 = 253.5 \text{ kip-ft}$$

$$M_{uy} = 1.058 \times 1.4 \times 75 = 111.1 \text{ kip-ft}$$

The interaction result, which may be considered a section efficiency ratio, is, from Eq. (7.39*a*) for $P_u/\phi P_n = 1513/4830 = 0.313 > 0.2$,

$$R = 0.312 + \frac{8}{9}\left(\frac{253.5}{3259} + \frac{111.1}{1628}\right)$$

$$= 0.313 + \frac{8}{9}(0.0778 + 0.682) = 0.443 < 1.0$$

Interaction Equation for Full Gravity Loading. For use in the interaction equation based on factored loads and moments due to 1.2 times the dead load plus 1.6 times the live load,

$$P_u = 1.2(750 + 325 + 0.426 \times 13) + 1.6 \times 250 = 1697 \text{ kips.}$$

Determined in the same way as for the dead load, the magnification factors are

$$B_{1x} = \frac{1.0}{1 - 1697/247{,}000} = 1.007$$

$$B_{1y} = \frac{1.0}{1 - 1697/27{,}700} = 1.065$$

Application of the magnification factors to the factored moments yields

$$M_{ux} = 1.007(1.2 \times 180 + 1.6 \times 75) = 338.4 \text{ kip-ft}$$

$$M_{uy} = 1.065(1.2 \times 75 + 1.6 \times 40) = 164.0 \text{ kip-ft}$$

With $P_u/\phi P_n = 1697/4830 = 0.351 > 0.2$, substitution of the preceding values in Eq. (7.39*a*) yields

$$R = 0.351 + \frac{8}{9}\left(\frac{338.4}{3259} + \frac{164.0}{1628}\right)$$

$$= 0.351 + \frac{8}{9}(0.1038 + 0.1008) = 0.533 < 1$$

Interaction Equation with Wind Load. For use in the interaction equation based on factored loads and moments due to 1.2 times the dead load plus 0.5 times the live load plus 1.3 times the wind load of 600 kips, including the $P - \Delta$ effect,

$$\begin{aligned} P &= 1.2(750 + 325 + 0.426 \times 13) + 0.5 \times 250 + 1.3 \times 600 \\ &= 2202 \text{ kips} \end{aligned}$$

Under wind action, double curvature may occur for strong-axis bending. For this condition, with $M_1 = M_2$,

$$C_{mx} = 0.6 - 0.4 \times 1 = 0.2$$

In this case, the magnification factor for strong-axis bending is

$$B_{1x} = \frac{0.2}{1 - 2202/247{,}000} = 0.202 < 1$$

Use $B_{1x} = 1.0$. The magnification factor for minor axis bending is, with $C_m = 1$ for single-curvature bending,

$$B_{1y} = \frac{1.0}{1 - 2202/27{,}700} = 1.0864$$

Application of the magnification factors to the factored moments yields

$$\begin{aligned} M_{ux} &= 1.0(1.2 \times 180 + 0.5 \times 75 + 1.3 \times 1050) \\ &= 1618 \text{ kip-ft} \\ M_{uy} &= 1.0864(1.2 \times 75 + 0.5 \times 40) = 119.5 \text{ kip-ft} \end{aligned}$$

With $P_u/\phi P_n^* = 2202/4830 = 0.456 > 0.2$, substitution of the preceding values in Eq. (7.39*a*) yields

$$R = 0.456 + \frac{8}{9}\left(\frac{1618}{3259} + \frac{119.5}{1628}\right)$$

$$= 0.456 + \frac{8}{9}(0.496 + 0.0734) = 0.96 < 1$$

This is the governing R value, and since it is less than unity, the column selected, W14 × 426, is adequate.

7.17 BASE PLATE DESIGN

Base plates are usually used to distribute column loads over a large enough area of supporting concrete construction that the design bearing strength of the concrete will not be exceeded. The factored load P_u is considered to be uniformly distributed under a base plate.

The nominal bearing strength f_p (ksi) of the concrete is given by

$$f_p = 0.85 f'_c \sqrt{A_1/A_1} \text{ and } \sqrt{A_2/A_1} \leq 2 \tag{7.41}$$

where f'_c = specified compressive strength of concrete, ksi
A_1 = area of the base plate, in^2
A_2 = area of the supporting concrete that is geometrically similar to and concentric with the loaded area, in^2

In most cases, the bearing strength f_p is $0.85 f'_c$ when the concrete support is slightly larger than the base plate or $1.7 f'_c$ when the support is a spread footing, pile cap, or mat foundation. Therefore, the required area of a base plate for a factored load P_u is

$$A_1 = P_u/\phi_c\, 0.85 f'_c \tag{7.42}$$

where ϕ_c is the strength reduction factor = 0.6. For a wide-flange column, A_1 should not be less than $b_f d$, where b_f is the flange width (in) and d is the depth of column (in).

The length N (in) of a rectangular base plate for a wide-flange column may be taken in the direction of d as

$$N = \sqrt{A_1} + \Delta > d \tag{7.43}$$

For use in Eq. (7.43),

$$\Delta = 0.5(0.95d - 0.80b_f) \tag{7.44}$$

The width B (in) parallel to the flanges, then, is

$$B = A_1/N \tag{7.45}$$

The thickness of the base plate t_p (in) is the largest of the values given by Eqs. (7.46) to (7.48):

$$t_p = m \sqrt{\frac{2P_u}{0.9F_y BN}} \tag{7.46}$$

$$t_p = n \sqrt{\frac{2P_u}{0.9F_y BN}} \tag{7.47}$$

$$t_p = \lambda n' \sqrt{\frac{2P_u}{0.9F_y BN}} \tag{7.48}$$

where m = projection of base plate beyond the flange and parallel to the web, in
= $(N - 0.95d)/2$
n = projection of base plate beyond the edges of the flange and perpendicular to the web, in
= $(B - 0.80b_f)/2$
$n' = \sqrt{(db_f)}/4$
$\lambda = (2\sqrt{X})/[1 + \sqrt{(1 - X)}] \leq 1.0$
$X = [(4\, db_f)/(d + b_f)^2][Pu/(\phi \times 0.85 f'_c A_1)]$

7.18 EXAMPLE—LRFD DESIGN OF COLUMN BASE PLATE

A base plate of A36 steel is to distribute the load from a W14 × 233 column to a concrete pedestal whose size is slightly larger than that of the base plate. The pedestal concrete strength f'_c is 4.0 ksi. The factored load on the column is 1731 kips.

From Eq. (7.41), the nominal bearing strength of the concrete is

$$f_p = 0.85 \times 4.0 = 3.4 \text{ ksi}$$

The required area of the base plate is computed from Eq. (7.42):

$$A_1 = 1{,}731/(0.6 \times 3.4) = 848.5 \text{ in}^2$$

From Eq. (7.44) for determination of the length N of the base plate,

$$\Delta = 0.5(0.95 \times 16.04 - 0.80 \times 15.89) = 1.26 \text{ in}$$

From Eq. (7.43),

$$N = \sqrt{848.5} + 1.26 = 30.4 \text{ in}$$

Use a 31-in length. The width required then is

$$B = A_1/N = 848.5/31 = 27.4 \text{ in}$$

Use a 28-in-wide plate. The area of the plate is $A_1 = 868$ in^2.

For determination of the base plate thickness, the projections beyond the column are

$$m = (31 - 0.95 \times 16.04)/2 = 7.88 \text{ in (governs)}$$

$$n = (28 - 0.8 \times 15.89)/2 = 7.64 \text{ in}$$

Equation (7.46) yields a larger thickness than Eq. (7.47) because $m > n$. From Eq. (7.46),

$$t_p = 7.88 \sqrt{\frac{2 \times 1731}{0.9 \times 36 \times 28 \times 41}} = 2.76 \text{ in}$$

For use in Eq. (7.48),

$$\begin{aligned}
n' &= \sqrt{(16.04 \times 15.89)}/4 = 4.0 \\
X &= [(4 \times 16.04 \times 15.89)/(16.04 + 15.89)^2][1731/(0.6 \times 0.85 \times 4.0 \times 868)] \\
&= 0.98 \\
\lambda &= (2 \times \sqrt{0.98})/[1 + \sqrt{(1 - 0.98)}] \\
&= 1.73 > 1.0
\end{aligned}$$

Then, the thickness given by Eq. (7.48) is

$$t_p = 4.0 \times 1.0 \sqrt{\frac{2 \times 1731}{0.9 \times 36 \times 868}} = 1.40 \text{ in} < 2.76 \text{ in}$$

Use a base plate 28 × 2¾ × 31 in.